KERALA CEE

Engineering Entrance Exam

Solved Paper 2020

Physics

1. A disc spinning at the rate 27.5 rad s^{-1} is slowed at the rate 10 rad s^{-2}. The time after which it will come to rest is

(a) 2.75 s (b) 5.5 s
(c) 1.25 s (d) 3.5 s
(e) 6.2 s

2. Four particles of masses, $m_1 = 1$ kg, $m_2 = 2$ kg, $m_3 = 1$ kg and m_4 are placed at the four corners of a square. The mass m_4 required, so that the centre of mass of all the four particles is exactly at the centre of the square is

(a) 3 kg (b) 4 kg
(c) 1.5 kg (d) 0.5 kg
(e) 2 kg

3. A solid sphere of radius r is revolving about one of its diameters with an angular velocity ω. If it suddenly expands uniformly, so that its radius increases to n times its original value, then its angular velocity becomes

(a) $n^2\omega$ (b) $\frac{\omega}{n^2}$ (c) $n\omega$ (d) $\frac{\omega}{n}$
(e) $2n\omega$

4. If a ring rolls down from top to bottom of an inclined plane, it takes time t_1. If it slides, it takes time t_2. Then, the ratio $\frac{t_2^2}{t_1^2}$ is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$
(e) $\frac{2}{5}$

5. If the distance between sun and earth is d, then the angular momentum of earth around the sun is proportional to

(a) $\sqrt{d}$ (b) d^2
(c) $d^{1/3}$ (d) d
(e) $d^{3/2}$

6. Two identical objects each of mass 50 kg are kept at a distance of separation of 50 cm apart on a horizontal table. The net gravitational force at the mid-point of the line joining their centres is

(a) zero (b) 6.6733×10^{-9}N
(c) 13.346×10^{-9}N (d) 3.336×10^{-9}N
(e) 6.673×10^{6}N

7. The ratio of the weight of a body at a height of $\frac{R}{10}$ from the surface of the earth to that at a depth of $\frac{R}{10}$ is (R is radius of earth)

(a) 4 : 5 (b) 1 : 1 (c) 9 : 8 (d) 2 : 3
(e) 8 : 9

8. Three thin wires of equal length are suspended from the top of a roof. The respective ratio of their area of cross-section is 1 : 2 : 4 and Young's modulii is 4 : 2 : 1, then the ratio of their weights to be attached at the other ends to obtain same elongation in them is

(a) 1 : 1 : 1 (b) 1 : 2 : 4
(c) 4 : 2 : 1 (d) $2 : \sqrt{2} : 1$
(e) $1 : \sqrt{2} : 2$

9. Water flows through a horizontal pipe of diameter 2 cm at a speed of 3 cm s^{-1}. The pipe has a nozzle of diameter 0.5 cm at its end. The speed of water emerging from the nozzle is

(a) 6 cm s^{-1} (b) 48 cm s^{-1}
(c) 16 cm s^{-1} (d) 12 cm s^{-1}
(e) 36 cm s^{-1}

10. The density of kerosene is 800 kg m^{-3}. Its relative density is

(a) 1.6 (b) 3.2 (c) 1 (d) 0.8
(e) 0.4

11. A solid sphere of volume V experiences a viscous force F when descending with a speed v in a liquid. If another solid sphere of volume 27 V descends with the same speed v in the same liquid, it experiences a viscous force

(a) 12 F (b) 6 F (c) 9 F (d) F
(e) 3 F

12. Two taps supply water to a container, one at the temperature of 20°C at the rate of 2 kg/min and another at 80°C at the rate of 1 kg/min. If the container gets water from the two taps simultaneously for 10 min, then the temperature of water in the container is

(a) 35°C (b) 30°C (c) 50°C (d) 40°C
(e) 45°C

13. If a monoatomic gas is compressed adiabatically to (1/27)th of its initial volume, then its pressure becomes

(a) 27 times (b) 125 times
(c) 243 times (d) 81 times
(e) 64 times

14. The values of C_p and C_V for a diatomic gas are respectively, (R = gas constant)

(a) $\frac{5}{2}R, \frac{7}{2}R$ (b) $\frac{3}{2}R, \frac{5}{2}R$
(c) $3R, 4R$ (d) $\frac{5}{2}R, \frac{3}{2}R$
(e) $\frac{7}{2}R, \frac{5}{2}R$

15. Three moles of an ideal gas are in a rigid cubical box with sides of length 0.170 m. The ratio of the forces that the gas exerts on each of the six sides of the box when the gas temperature are 27°C and 127°C is

(a) 6 : 1 (b) 1 : 2 (c) 3 : 1 (d) 3 : 4
(e) 1 : 3

16. The average kinetic energy of a monoatomic gas molecule kept at temperature 27°C is (Boltzmann constant, $k = 1.3 \times 10^{-23}$ JK^{-1})

(a) 5.85×10^{-21} J (b) 4.12×10^{-21} J
(c) 3.75×10^{-21} J (d) 2.85×10^{-21} J
(e) 7.55×10^{-21} J

17. A travelling wave in a medium is given by the equation, $y = a\sin(\omega t - kx)$. The maximum acceleration of the particle in the medium is

(a) $a\omega$ (b) $a\omega^2$ (c) $\frac{\omega}{k}$ (d) $\frac{x}{t}$
(e) $k\omega$

18. Two simple harmonic motions with the same amplitude and same frequency acting in the same direction are impressed on a particle. If the resultant amplitude of the particle is equal to the amplitude of individual SHMs, the phase difference between the two simple harmonic motions, is

(a) $\frac{2\pi}{\sqrt{3}}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{2\pi}{3}$
(e) $\frac{\pi}{3}$

19. The nearest harmonics of an organ pipe open at both the ends are 200 Hz and 240 Hz. The fundamental frequency is

(a) 40 Hz (b) 20 Hz (c) 30 Hz (d) 80 Hz
(d) 50 Hz

20. Two strings of the same material and same length are given equal tension. If they are vibrating with fundamental frequencies 1600 Hz and 900 Hz, then the ratio of their respective diameters, is

(a) 16 : 9 (b) 4 : 3 (c) 81 : 256 (d) 3 : 4
(e) 9 : 16

21. An object, moving in a straight line with velocity 100 ms^{-1}, goes past a stationary observer. If the object emits note of 400 Hz while moving, the change in the frequency noted by the observer as the object goes past him is (speed of sound in air = 300 ms^{-1})

(a) 350 Hz (b) 300 Hz
(c) 200 Hz (d) 100 Hz
(e) 150 Hz

22. The electric flux (in SI units) through any face of a cube due to a positive charge Q situated at the centre of a cube is

(a) $\frac{Q}{4\pi\varepsilon_0}$ (b) $4\pi\varepsilon_0 Q$ (c) $\frac{Q}{6\varepsilon_0}$ (d) $\frac{Q}{6\pi\varepsilon_0}$
(e) $6\pi\varepsilon_0 Q$

23. A capacitance of a parallel plate air capacitor is $10\mu F$. Dielectric constant of the medium to be introduced in between its plates to double its capacitance is

(a) 2 (b) 3
(c) 4 (d) 2.5
(e) 1.5

24. The electric potential V at any point (x, y, z) in space is given by $V = 4z^2$ V, where x, y, z are all in metre. The electric field at that point (1m, 0, 2m) in $V\ m^{-1}$ is

(a) 16 along the positive Z-axis
(b) 16 along the negative Z-axis
(c) 4 along the positive Z-axis
(d) 4 along the negative Z-axis
(e) 8 along the negative Z-axis

25. The work done in moving a point charge of $10\mu C$ through a distance of 3 cm along the equatorial axis of an electric dipole is

(a) 10×10^{-6} J (b) 30×10^{-6} J
(c) 20×10^{-6} J (d) 5×10^{-6} J
(e) zero

26. A steady current flows in a metallic conductor of non-uniform cross-section. The quantity/quantities that remains/remain constant along the length of the conductor is/are

(a) current, electric field and drift speed
(b) drift speed only
(c) current and drift speed
(d) current and electric field
(e) current only

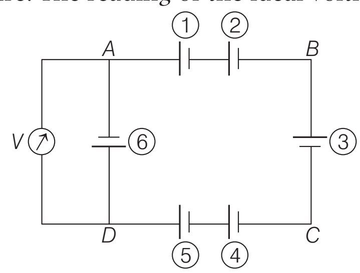
27. In a platinum resistance thermometer, the resistances of the wire at ice point and steam point are of 4 Ω and 4.25 Ω, respectively. When the thermometer is kept in a hot water bath, whose temperature is not known, the resistance of the wire is found to be 4.5 Ω. The temperature of the hot water bath is

(a) 150°C (b) 100°C (c) 300°C (d) 350°C
(e) 200°C

28. Internal resistance of a cell is independent of

(a) the circuit elements connected to it
(b) surface area of the electrode
(c) distance between the electrode
(d) concentration of the electrolytes
(e) temperature of the electrolytes

29. Six cells, each of emf 5V and internal resistance 0.1 Ω are connected as shown in figure. The reading of the ideal voltmeter V is

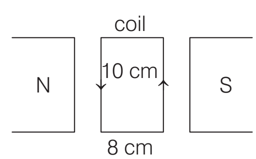


(a) 30 V (b) 5 V (c) 15 V (d) zero
(e) 0.5 V

30. Which one of the following characteristics is not associated with a paramagnetic material?

(a) It is weakly magnetised in the direction of the magnetising field, in which it is placed.
(b) Its magnetic permeability is greater than one.
(c) Its magnetic susceptibility is positive.
(d) Its magnetic susceptibility increases with rise in temperature.
(e) Its individual atom/molecule/ion has a net non-zero magnetic moment of its own.

31. A coil of 50 turns carrying a current of 2A in a magnetic field of 0.5 T. The torque acting on the coil is



(a) 0.4 N-m clockwise (b) 0.2 N-m anti-clockwise
(c) 0.4 N-m anti-clockwise (d) 0.2 N-m clockwise
(e) 0.8 N-m anti-clockwise

32. A long solenoid with 500 turns per unit length carries a current of 1.5 A. The magnetic induction at one of the ends of the solenoid on its axis is nearly

(a) 32×10^{-4} T (b) 4×10^{-5} T
(c) 47×10^{-5} T (d) 16×10^{-4} T
(e) 8×10^{-5} T

33. Choose the **wrong** statement.

(a) The magnetic declination is greater at higher altitudes and smaller near the equator.
(b) In most of the northern hemisphere, the south pole of the dip needle tilts downwards.
(c) Circulating electron in an atom has a magnetic moment.
(d) The magnetic declination at Delhi is $0^\circ 41'$E and at Mumbai is $0^\circ 58'$W.
(e) At the poles, the magnetic field lines are converging or diverging vertically, so that the horizontal component is negligible.

34. The magnetic field at the centre of a circular coil of 50 turns and radius 10 cm carrying a current of 1A, (in T) is

(a) $\pi \times 10^{-4}$ (b) $\pi \times 10^{-2}$
(c) $2\pi \times 10^{-3}$ (d) $\frac{\pi}{4} \times 10^{-5}$
(e) $\frac{\pi}{2} \times 10^{-4}$

35. Choose the **wrong** statement for the pure inductive circuit.

(a) The inductive reactance limits the current in a purely inductive circuit.
(b) The average power supplied to an inductor over one complete cycle is zero.
(c) The inductive reactance is directly proportional to the frequency of the current.
(d) The emf of the source and current oscillates symmetrically about zero value.
(e) The current leads the voltage by $\frac{\pi}{2}$.

36. A train is running at a speed of 72 km h^{-1} on the rails separated by a distance of 150 cm. If the vertical component of earth's magnetic field at the place is 4.0×10^{-5}T. The induced emf on the rails, is

(a) 1.2 mV (b) 3 mV
(c) 2.5 mV (d) 0.5 mV
(e) 4.2 mV

37. A transformer operates at $V_p = 6$ kV on the primary side and supplies electric energy at $V_s = 220$ V to a number of houses in a town. If the total power consumption of the town is 7.2 kW, the current (in A) in the primary is

(a) 2 (b) 1.2 (c) 2.5 (d) 3
(e) 1

38. The relation between the charge flow ΔQ through the circuit of resistance r and the change in the magnetic flux $\Delta\phi_B$ is

(a) $\Delta Q = \frac{\Delta\phi_B}{r}$ (b) $\Delta\phi_B = \frac{\Delta Q}{r}$
(c) $\Delta\phi_B = \Delta Q$ (d) $\Delta\phi_B = \frac{\Delta Q}{r^2}$
(e) $\Delta Q = \frac{r}{\phi_B}$

39. If an electromagnetic wave of frequency 5 MHz travels from vacuum into a dielectric medium of electrical permittivity $\varepsilon_r = 4$, then its (take, $\mu_r = 1$)

(a) wavelength is halved and the frequency remains unchanged
(b) wavelength and frequency are both doubled
(c) wavelength and frequency both remain unchanged
(d) wavelength is doubled but the frequency remains unchanged
(e) wavelength remains unchanged but the frequency is doubled

40. Among the following, which is not true for ultraviolet light?

(a) Induces the production of more melanin, causing tanning of the skin.
(b) Can be focused into very narrow beams.
(c) Kills germs in water purifiers.
(d) Used in eye surgery.
(e) Treatment for certain forms of cancer.

41. Choose the **wrong** statement.

(a) A ray entering a material of larger index of refraction bends toward the normal.
(b) A ray entering a material of smaller index of refraction bends away from the normal.
(c) A ray oriented along the normal does not bend, regardless of the materials.
(d) Light rays from any submerged object bend away from the normal when they emerge into the air.
(e) When a wave passes from one material into a second material with larger index of refraction, the wave speed increases.

42. Angular width of the first minimum on either side of the central maximum due to a single slit of width a, illuminated by a light of wavelength λ is

(a) $\frac{\lambda}{a}$ (b) $\frac{\lambda}{2a}$ (c) $\frac{2\lambda}{a}$ (d) $\frac{\lambda}{4a}$
(e) $\frac{4\lambda}{a}$

43. The reflected ray is completely polarised for certain angle of incidence in a transparent medium. If the angle of refraction is 30°, then the refractive index of the medium is

(a) 1.5 (b) 1.732 (c) 1.33 (d) 1.414
(e) 1.6

44. A certain prism produces a minimum deviation of 42°. It produces a deviation of 45° when the angle of incidence is either 43° or 62°. The angle of incidence when the prism undergoes minimum deviation is

(a) 60° (b) 30° (c) 49° (d) 51°
(e) 40°

45. If two waves of intensities I and $4I$ superpose, the ratio between maximum and minimum intensities, is

(a) 9 : 1 (b) 5 : 2
(c) 4 : 3 (d) 3 : 1
(e) 6 : 1

46. Among the following photosensitive substances, the one which emits electrons when it is illuminated by visible light is

(a) magnesium (b) zinc
(c) sodium (d) cadmium
(e) platinum

47. The de-Broglie wavelength of the matter wave associated with an object dropped from a height x, when it reaches the ground is proportional to

(a) x^2 (b) $\frac{1}{\sqrt{x}}$
(c) $\sqrt{x}$ (d) $x^{3/2}$
(e) x

48. The number of α-particles emitted during the radioactive decay chain from $^{226}_{88}Ra$ and ending at $^{206}_{82}Pb$ is

(a) 5 (b) 4 (c) 6 (d) 3
(e) 2

49. The shortest wavelength of Paschen series in hydrogen spectrum is 8182 Å. The first member of the Paschen series is nearly

(a) 15400 Å (b) 12200 Å
(c) 13400 Å (d) 18700 Å
(e) 16700 Å

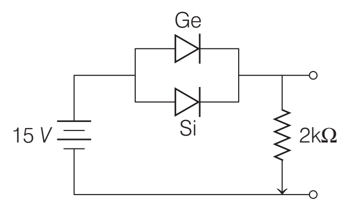
50. A nucleus, initially at rest, breaks up into two nuclear fragments with their radii in the ratio 2 : 1. Then, their velocities will be in the ratio

(a) 3 : 2 (b) 1 : 5 (c) 1 : 8 (d) 2 : 1
(e) 1 : 4

51. The ratio of the energy released by 4 kg of hydrogen at sun by fusion process to 23.5 kg of ^{235}U in the nuclear reactor by fission process is (assume energy released per fusion is 26 MeV and that per fission is 200 MeV)

(a) 5 : 13 (b) 1 : 26 (c) 13 : 10 (d) 10 : 13
(e) 26 : 1

52. If the Ge-diode in the circuit is reverse biased, the current through 2 kΩ resistor



(a) increases by 0.2 mA (b) decreases by 0.4 mA
(c) increases by 0.4 mA (d) decreases by 0.25 mA
(e) does not change

53. The contribution to the total current in a semiconductor, due to electrons and holes are 0.75 and 0.25, respectively. The drift velocity of electrons is 3/2 times that of holes at this temperature. Then, the ratio between electron concentration and hole concentration is

(a) 1 : 3 (b) 3 : 2 (c) 6 : 5 (d) 4 : 1
(e) 2 : 1

54. In a common emitter amplifier, the input resistance and output resistance are 200 Ω and 500 Ω respectively. If the voltage gain of the amplifier is 50, then the power gain is

(a) 1250 (b) 1000 (c) 750 (d) 100
(e) 500

55. The gates that give output $Y = 0$ for the two inputs $A = 1$ and $B = 1$ are
(a) AND and OR gates
(b) OR, AND and NAND gates
(c) NOR and OR gates
(d) NOR and NAND gates
(e) NAND and AND gates

56. In amplitude modulation of audio frequency 700 Hz, the appropriate carrier frequency to be used is
(a) 5 MHz (b) 50 MHz
(c) 1000 kHz (d) 350 kHz
(e) 1000 MHz

57. The maximum line-of-sight distance d_M between the transmitting antenna of height h_T and receiving antenna of height h_R in LOS communication is (R = radius of the earth)
(a) $h_T + h_R$ (b) $\sqrt{h_T + h_R}$
(c) $\frac{h_T + h_R}{2}$ (d) $\sqrt{h_T} + \sqrt{h_R}$
(e) $\sqrt{2Rh_T} + \sqrt{2Rh_R}$

58. If ε_0 and μ_0 are respectively the electrical permittivity and magnetic permeability of vacuum, the dimensional formula for $\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ is
(a) [MLT] (b) [MLT^{-2}]
(c) [$ML^{-1}T^{-1}$] (d) [M^0LT^{-1}]
(e) [$M^0L^{-2}T$]

59. The power in an electrical circuit for a current of 5 ± 0.4 A and voltage 10 ± 0.2 V is measured at 10% error. To measure the power at 5% error the current should be measured at an error of
(a) 5% (b) 2% (c) 10% (d) 3%
(e) 4%

60. The angular diameter of a planet measured from earth is 90″. If the diameter of the planet is $\pi \times 10^6$ m, then its distance from the earth is
(a) 3.6×10^9 m (b) 7.2×10^9 m
(c) 3.6×10^6 m (d) 7.2×10^6 m
(e) 1.8×10^8 m

61. The angle between **A** and the resultant of $2\mathbf{A} + 3\mathbf{B}$ and $4\mathbf{A} - 3\mathbf{B}$ is
(a) 90° (b) $\tan^{-1}\left(\frac{A}{B}\right)$
(c) $\tan^{-1}\left(\frac{B}{A}\right)$ (d) $\tan^{-1}\left(\frac{A - B}{A + B}\right)$
(e) 0°

62. A particle is moved in a semi-circular path of radius R. Then,
(a) its average velocity is zero
(b) its average acceleration is zero
(c) its magnitude of displacement is $2R$
(d) its average velocity and average speed are equal
(e) its distance travelled is equal to displacement

63. Two projectiles P and Q thrown with velocities v and $\frac{v}{2}$ respectively, have the same range. If Q is thrown at an angle of 15° to the horizontal, P must be thrown at an angle of
(a) 30° (b) $\frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$
(c) $\frac{1}{4}\sin^{-1}\left(\frac{1}{2}\right)$ (d) 60°
(e) 45°

64. An object is thrown vertically with a velocity u. The velocity with which it strikes the ground on its return is
(a) $\frac{u}{2}$ (b) $\frac{-u}{2}$
(c) $-u$ (d) u
(e) $2u$

65. Choose the **correct** statement.
(a) Second law of motion is a vector equation.
(b) Second law of motion is applicable to a particle and not to the system of particles.
(c) Force is always in the direction of motion.
(d) If external force on a body is zero, it does not mean the acceleration is zero.
(e) Acceleration at an instant depends on the history of the motion of the particle.

66. A boy is standing on a weighing machine inside a lift. When the lift goes upwards with acceleration $\frac{g}{4}$, the machine shows the reading 50 kg-wt When the lift goes downward with acceleration $\frac{g}{4}$, the reading of the machine in kg-wt would be

(a) 50 (b) 30 (c) 45.5 (d) 62.5
(e) 14

67. A ship of mass 2×10^7 kg initially at rest is pulled by a force of 5×10^5N through a distance of 2m. Assuming that the resistance due to water is negligible, the speed of the ship is

(a) 2 ms^{-1} (b) 0.01 ms^{-1}
(c) 0.1 ms^{-1} (d) 1 ms^{-1}
(e) 5 ms^{-1}

68. A force of $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ N acts on a body of mass 1 kg which is at rest initially. The acceleration of the body is

(a) $(4\hat{i} + 6\hat{j}) \text{ ms}^{-2}$ (b) $(2\hat{i} + 3\hat{j}) \text{ ms}^{-2}$
(c) $(3\hat{i} + 5\hat{j}) \text{ ms}^{-2}$ (d) $(6\hat{i} + 2\hat{j}) \text{ ms}^{-2}$
(e) $(\hat{i} + \hat{j}) \text{ ms}^{-2}$

69. The work-energy theorem,

(a) does not hold in all inertial frames
(b) is independent of Newton's second law
(c) may be viewed as a scalar form of Newton's second law
(d) cannot be extended to non-inertial frames
(e) is independent of Newton's third law

70. A running boy has the same kinetic energy as that of a man of twice his mass. If the speed of the boy is 14.14 ms^{-1}, the speed of the man is

(a) 1.414 ms^{-1} (b) 0.25 ms^{-1}
(c) 10 ms^{-1} (d) $3\sqrt{2} \text{ ms}^{-1}$
(e) 0.5 ms^{-1}

71. A body of mass 2 kg is moving with a momentum of 10 kg ms^{-1}. The force needed to increase its kinetic energy by four times in 10 s is

(a) 2 N (b) 4 N (c) 1 N (d) 0.5 N
(e) 8 N

72. If a force $\mathbf{F} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ acting on a particle displaces it from (1, 1, 1) to (2, − 1, 0), then the work done by the force (in units of work) is

(a) 2 (b) 1 (c) 5 (d) 4
(e) 9

Chemistry

73. In which one of the following reactions, entropy decreases?

(a) Sodium chloride is dissolved in water
(b) Water is heated from 303K to 353K
(c) Sodium bicarbonate is decomposed to $Na_2CO_3(s)$, $CO_2(g)$ and $H_2O(g)$
(d) Water crystallises into ice
(e) Dihydrogen molecule is decomposed into hydrogen atoms

74. The standard enthalpies of formation of $H_2O(l)$ and $CO_2(g)$ are respectively -286 kJ mol^{-1} and -394 kJ mol^{-1}. If the standard heat of combustion of $CH_4(g)$ is -891 kJ mol^{-1}, then the standard enthalpy of formation of $CH_4(g)$ is

(a) -75 kJ mol^{-1} (b) $+75 \text{ kJ mol}^{-1}$
(c) -211 kJ mol^{-1} (d) $+211 \text{ kJ mol}^{-1}$
(e) $-1571 \text{ kJ mol}^{-1}$

75. The equilibrium constant for the equilibrium $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ at a particular temperature is $2 \times 10^{-2} \text{ mol dm}^{-3}$. The number of moles of PCl_5 that must be taken in a one litre flask at the same temperature to obtain a concentration of 0.20 mol of chlorine at equilibrium is

(a) 2.0 (b) 2.2
(c) 1.8 (d) 0.2
(e) 0.1

76. The pH of the resultant solution obtained by mixing 20 mL of 0.01M HCl and 20 mL of 0.005M $Ca(OH)_2$ is

(a) 2 (b) 0
(c) 1 (d) 7
(e) 5

77. $CH_4(g) + 4Cl_2(g) \longrightarrow CCl_4(l) + 4HCl(g)$
In the above reaction, the change of oxidation state of carbon is
(a) from + 4 to – 4 (b) from + 1 to + 4
(c) from – 4 to + 4 (d) from – 1 to + 1
(e) from – 4 to – 1

78. How many moles of platinum will be deposited on the cathode when 0.40 F of electricity is passed through a 1.0M solution of Pt^{4+}?
(a) 0.60 mol (b) 1.0 mol
(c) 0.40 mol (d) 0.45 mol
(e) 0.10 mol

79. When the same amount of the solute '*P*' and '*Q*' are separately dissolved in 500g water the ΔT_f values are 0.15 K and 0.30 K respectively. If the molecular weight of '*P*' is 80 g mol^{-1}, then the molecular weight of '*Q*' is
(a) 30 g mol^{-1} (b) 60 g mol^{-1}
(c) 40 g mol^{-1} (d) 45 g mol^{-1}
(e) 160 g mol^{-1}

80. A solution is prepared by dissolving 20g NaOH in 1250 mL of a solvent of density 0.8 g/mL. Then the molality of the solution is
(a) 0.2 mol kg^{-1} (b) 0.08 mol kg^{-1}
(c) 0.25 mol kg^{-1} (d) 0.0064 mol kg^{-1}
(e) 0.5 mol kg^{-1}

81. The rate constant of a first order reaction is $231 \times 10^{-5}\ s^{-1}$. How long will 4g of this reactant reduce to 2g?
(a) 310 s (b) 300 s (c) 210 s (d) 30.1 s
(e) 230.3 s

82. An endothermic reaction $A \rightarrow B$ has an activation energy of 13 kJ mol^{-1} and the enthalpy change for the reaction is 2 kJ mol^{-1}. The activation energy of the reaction $B \rightarrow A$ is
(a) 15 kJ mol^{-1} (b) 11 kJ mol^{-1}
(c) 2 kJ mol^{-1} (d) – 15 kJ mol^{-1}
(e) 26 kJ mol^{-1}

83. Adsorption is accompanied by
(a) decrease in enthalpy and decrease in entropy
(b) increase in enthalpy and decrease in entropy
(c) decrease in enthalpy and increase in entropy
(d) increase in enthalpy and increase in entropy
(e) no change in enthalpy and entropy

84. In the coagulation of a positive sol, the flocculating power of the ions PO_4^{3-}, SO_4^{2-} and Cl^- decreases in the order
(a) $PO_4^{3-} > Cl^- > SO_4^{2-}$ (b) $PO_4^{3-} > SO_4^{2-} > Cl^-$
(c) $Cl^- > SO_4^{2-} > PO_4^{3-}$ (d) $Cl^- > PO_4^{3-} > SO_4^{2-}$
(e) $SO_4^{2-} > PO_4^{3-} > Cl^-$

85. Which one of the following nitrates does not give the corresponding metallic oxide, nitrogen dioxide and oxygen on heating?
(a) Lithium nitrate (b) Beryllium nitrate
(c) Magnesium nitrate (d) Calcium nitrate
(e) Potassium nitrate

86. Which of the following statement is incorrect about beryllium?
(a) Beryllium hydroxide is amphoteric.
(b) Beryllium compounds are largely covalent.
(c) Beryllium is not easily attacked by acids.
(d) Beryllium exhibit coordination number of 4.
(e) Beryllium hydroxide dissolves in excess of alkali to give a beryllate ion.

87. The oxyacid of phosphorus that contains one P—OH, two P—H and one P $=$ O bonds is
(a) phosphinic acid (b) phosphoric acid
(c) pyrophosphoric acid (d) hypophosphoric acid
(e) pyrophosphorus acid

88. Choose the correct statements about diborane.
I. It is prepared by the oxidation of sodium borohydride with iodine.
II. It undergoes cleavage reactions with Lewis bases to give borane adducts.
III. It is produced on an industrial scale by the reaction of BF_3 with $LiAlH_4$.
IV. It is readily hydrolysed by water to give borazine.
V. It burns in oxygen and gives boron trioxide.
(a) I, II, III (b) I, II, V
(c) I, II, IV (d) II, III, IV
(e) I, III, V

89. Which one of the following actinoid has no electron in 6*d*-orbital?
(a) Pa (b) Np (c) Lr (d) Cm
(e) Pu

90. The catalyst used in the Wacker process of oxidation of ethene to ethanal is
(a) silver (b) nickel
(c) $PdCl_2$ (d) V_2O_5
(e) *Ziegler* catalyst

91. The correct formula of dichlorobis (triphenylphosphine) nickel (II) is
(a) $[NiCl_2(PPh_3)_2]Cl$ (b) $[NiCl_2(PPh_3)]$
(c) $[NiCl_2(PPh_2)_3]$ (d) $[NiCl(PPh_3)_2]Cl$
(e) $[NiCl_2(PPh_3)_2]$

92. Which one of the following is an ambidentate ligand?
(a) Cl^- (b) H_2O
(c) $H_2NCH_2CH_2NH_2$ (d) SCN^-
(e) $C_2O_4^{2-}$

93. Which one is not correctly matched?

Ore	Composition
(a) Siderite	$FeCO_3$
(b) Calamine	$ZnCO_3$
(c) Sphalerite	ZnS
(d) Kaolinite	$[Al_2(OH)_4Si_2O_5]$
(e) Cuprite	$CuCO_3.Cu(OH)_2$

94. Which one of the following is a benzenoid aromatic compound?
(a) Cyclooctatetraene (b) Hexyne
(c) Cyclohexane (d) Toluene
(e) Cyclopentadiene

95. The products obtained by the ozonolysis of 2-methylbut-l-ene are
(a) propanone and ethanal
(b) propanone and methanal
(c) butanone and methanal
(d) ethanal and propanal
(e) butanone and methanol

96. Which one of the following is not an isomer of 3-methylbut-1-yne?
(a) 2, 3-dimethylbuta-1, 3-diene
(b) Pent-1-yne
(c) Pent-2-yne
(d) Penta-1, 3-diene
(e) 2-methylbuta-1, 3-diene

97. The compound that does not undergo hydrolysis by S_N1 mechanism is
(a) $C_6H_5CH_2Cl$ (b) $C_6H_5CH(CH_3)Cl$
(c) C_6H_5Cl (d) CH_3CH_2Cl
(e) $C_6H_5CH(C_6H_5)Cl$

98. Which one of the following is a secondary alcohol?
(a) 2-methylbutan-2-ol (b) 3-methylbutan-1-ol
(c) 2-methylbutan-1-ol (d) 3-methylbutan-2-ol
(e) 2, 2-dimethylbutan-1-ol

99. An organic compound '*A*' with molecular formula C_7H_6O forms 2, 4-DNP derivative and reduces Tollen's reagent. When '*A*' is heated with conc. KOH, it gives sodium benzoate and compound '*B*'. The compound '*B*' is
(a) benzene (b) toluene
(c) acetophenone (d) benzaldehyde
(e) benzyl alcohol

100. Which one of the following compounds would undergo Cannizzaro reaction?
(a) 2-methylpentanal (b) Cyclohexanone
(c) 2, 2-dimethylbutanal (d) 1-phenylpropanone
(e) Phenylacetaldehyde

101. Which one of the following can be prepared by Gabriel phthalimide synthesis?
(a) 2-aminotoluene (b) Aniline
(c) 4-bromoaniline (d) Allyl amine
(e) N-methylethanamine

102. The reagent that is used to distinguish between a secondary amine and a tertiary amine is
(a) *p*-toluene sulphonyl chloride
(b) dil.HCl
(c) dil.NaOH
(d) $CHCl_3$ and alc.KOH
(e) bromine water

103. Choose the correct statement of the following.
(a) Cellulose is also known as animal starch.
(b) A linkage between two monosaccharide units through oxygen atom is called oxide linkage.
(c) Glucose on oxidation with bromine water gives *n*-hexane.
(d) Carbohydrates are used as storage molecules as starch in animals.
(e) Water insoluble component of starch is amylopectin.

104. Among the following which one is a non-reducing sugar?
(a) Lactose (b) Glucose
(c) Sucrose (d) Maltose
(e) Fructose

105. Which one of the following polymer is a copolymer formed by condensation polymerisation?
(a) Buna-S
(b) Neoprene
(c) Polythene
(d) Melamine-formaldehyde
(e) Buna-N

106. Which one of the following sets forms the biodegradable polymer?
(a) 3-hydroxybutanoic acid and 3-hydroxypentanoic acid
(b) Acrylonitrile and 1, 3-butadiene
(c) Urea and formaldehyde
(d) Ethylene glycol and terephthalic acid
(e) Adipic acid and hexamethylene diamine

107. The antimicrobial drug that contains arsenic is
(a) prontosil (b) salvarsan
(c) sulphapyridine (d) ofloxacin
(e) sulphanilamide

108. Which one of the following statements is not correct?
(a) All monosaccharides are reducing sugars.
(b) Lactose is commonly known as milk sugar.
(c) Glucose pentaacetate does not react with hydroxylamine.
(d) Glucose does not give 2,4-DNP test.
(e) Glucose on oxidation with bromine water, gives saccharic acid.

109. Which one of the following is an antifertility drug?
(a) Bithionol (b) Ofloxacin
(c) Norethindrone (d) Aspartame
(e) Terpineol

110. Which one of the following is a greenhouse gas?
(a) Methane (b) Ethane
(c) Hydrogen sulphide (d) Acetylene
(e) Ethylene

111. Which one of the following will have the largest number of atoms?
(a) 1g Au(*s*) (b) 1g Na(*s*)
(c) 1g Li(*s*) (d) 1g of $Cl_2(g)$
(e) 1g of $O_2(g)$

112. An organic compound contains 24% carbon, 4% hydrogen and remaining chlorine. Its empirical formula is
(a) CHCl (b) CH_2Cl
(c) $CHCl_2$ (d) CH_3Cl
(e) CH_2Cl_2

113. The IUPAC name of an element is unbinilium. Its atomic number is
(a) 102 (b) 110 (c) 120 (d) 106
(e) 100

114. The number of electrons, protons and neutrons in a species are equal to 10, 11 and 12 respectively. The proper symbol of the species is
(a) ${}^{22}_{11}Na^{+}$ (b) ${}^{23}_{11}Na$
(c) ${}^{23}_{10}Ne$ (d) ${}^{23}_{11}Na^{+}$
(e) ${}^{23}_{11}Na^{2+}$

115. Which one of the following element is represented as Eka-silicon in Mendeleev's periodic table?
(a) Gallium (b) Germanium
(c) Aluminium (d) Tin
(e) Arsenic

116. The correct match among the following is

(A)	Lithium, sodium, potassium	(I)	Alkaline earth metals
(B)	Beryllium, magnesium, calcium	(II)	Semi-metals
(C)	Oxygen, sulphur, selenium	(III)	Alkali metals
(D)	Silicon, germanium, arsenic	(IV)	Chalcogens

(a) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(b) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(c) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
(d) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(e) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

117. Which one of the following molecules is formed by sp^3d hybridisation?
(a) BrF_5 (b) PF_5
(c) SF_6 (d) $[Co(NH_3)_6]^{3+}$
(e) $[Pt(Cl)_4]^{2-}$

118. The correct order of bond energy (in kJ/mol) of the following molecules is

(a) $O_2 < B_2 < C_2 < N_2$ (b) $B_2 < C_2 < O_2 < N_2$
(c) $C_2 < O_2 < B_2 < N_2$ (d) $B_2 < O_2 < C_2 < N_2$
(e) $B_2 < O_2 < N_2 < C_2$

119. The type of attractive forces that operate between gaseous HCl molecules is

(a) dipole-dipole forces
(b) dispersion forces
(c) ion-dipole forces
(d) dipole-induced dipole forces
(e) electrostatic forces

120. Schottky defect is shown by

(a) ionic substances in which the size of the cation is smaller than that of the anion
(b) ionic substances in which the cation and anion are of almost similar sizes
(c) ionic substances in which the size of the cation is larger than that of the anion
(d) non-stoichiometric inorganic solids
(e) non-ionic substances

Mathematics

1. The domain of the function f given by $f(x) = \sqrt{x-1}$ is

(a) $(-\infty, \infty)$ (b) $(1, \infty)$
(c) $[1, \infty)$ (d) $[0, \infty)$
(e) $(0, \infty)$

2. Let $f(x) = -2x^2 + 1$ and $g(x) = 4x - 3$, then $(gof)(-1)$ is equal to

(a) 9 (b) – 9
(c) 7 (d) – 7
(e) – 8

3. Let A and B be finite sets such that $n(A - B) = 18$, $n(A \cap B) = 25$ and $n(A \cup B) = 70$. Then $n(B)$ is equal to

(a) 52 (b) 25
(c) 27 (d) 43
(e) 45

4. In a group of 100 persons, 80 people can speak Malayalam and 60 can speak English. Then the number of people who speak English only is

(a) 40 (b) 30
(c) 20 (d) 25
(e) 35

5. If * is a binary operation defined by $a * b = \frac{a}{b} + \frac{b}{a} + \frac{1}{ab}$ for positive integers a and b, then $2 * 5$ is equal to

(a) 4 (b) 3 (c) 2 (d) 1
(e) 5

6. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$, then $A - B =$

(a) {1, 3, 5, 6} (b) {0, 1, 3, 5, 6}
(c) {1, 3, 5} (d) {1, 2, 3, 4, 5, 6}
(e) {2, 4}

7. Let $A = \{2, 3, 4, 5\}$, $B = \{36, 45, 49, 60, 77, 90\}$ and let R be the relation 'is factor of' from A to B. Then the range of R is the set

(a) {60} (b) {36, 45, 60, 90}
(c) {49, 77} (d) {49, 60, 77}
(e) {36, 45, 49, 60, 77, 90}

8. The real part of $e^{(3+4i)x}$ is

(a) e^{3x} (b) $\cos 7x$
(c) $e^{3x} \cos 4x$ (d) $e^{3x} \sin 4x$
(e) 0

9. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\frac{1}{p^2 + q^2}\left(\frac{x}{p} + \frac{y}{q}\right)$ is equal to

(a) – 2 (b) – 1 (c) 1 (d) 2
(e) 0

10. Let $z = x + iy$ be a complex number such that $|z + i| = 2$. Then the locus of z is a circle whose centre and radius are

(a) (0, – 1) ; 2 (b) (0, 2) ; 2
(c) (1, – 1) ; 2 (d) (0, – 1) ; $\sqrt{3}$
(e) (0, 2) ; $\sqrt{3}$

11. If $2 + i$ is a root of $x^2 - 4x + c = 0$, where c is a real number, then the value of c is

(a) 2 (b) 3 (c) 4 (d) 5
(e) 0

12. Let z_1 and z_2 be complex numbers satisfying $|z_1| = |z_2| = 2$ and $|z_1 + z_2| = 3$. Then $\left|\frac{1}{z_1} + \frac{1}{z_2}\right| =$

(a) $\frac{3}{2}$ (b) 2 (c) $\frac{3}{4}$ (d) $\frac{1}{2}$
(e) 4

13. The principal argument of the complex number $z = \frac{1 + \sin\pi - i\cos\pi}{1 + \sin\pi + i\cos\pi}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{5}$ (d) $\frac{\pi}{2}$
(e) $\frac{\pi}{4}$

14. If $z_1 = 2 + 3i$ and $z_2 = 3 + 2i$, then $|z_1 + z_2|$ is equal to

(a) 50 (b) 10 (c) $5\sqrt{2}$ (d) 25
(e) $2\sqrt{5}$

15. $\frac{10i}{1 + 2i}$ is equal to

(a) $-2i$ (b) $2i$ (c) $-4 + 2i$ (d) $4 + 2i$
(e) $6i$

16. The value of $\sum_{k=1}^{10} (3k^2 + 2k - 1)$ is

(a) 1120 (b) 1200 (c) 1230 (d) 1265
(e) 1255

17. The number $a_1, a_2, a_3, \ldots$ form an arithmetic sequence with $a_1 \neq a_2$. The three numbers a_1, a_2 and a_6 form a geometric sequence in that order. Then the common difference of the arithmetic sequence is

(a) a_1 (b) $2a_1$ (c) $3a_1$ (d) $4a_1$
(e) $5a_1$

18. In an arithmetic sequence, the sum of first and third terms is 6 and the sum of second and fourth terms is 20. Then the 11th term is

(a) 67 (b) 62 (c) 57 (d) 73
(e) 66

19. In an AP, the first term is 3 and the last term is 17. The sum of all the terms in the sequence is 70. Then the number of terms in the arithmetic sequence is

(a) 7 (b) 5 (c) 9 (d) 6
(e) 8

20. Consider the set of all positive rational numbers that are less than 1 and that have denominators as 30 in their lowest terms. Their sum is equal to

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

21. If p, q and 23 is an increasing arithmetic sequence and p and q are prime numbers, then $p + q =$

(a) 22 (b) 24 (c) 26 (d) 28
(e) 30

22. The 5th and 7th terms of a GP are 12 and 48 respectively. Then the 9th term is

(a) 162 (b) 96 (c) 192 (d) 144
(e) 182

23. The number of positive integers less than 1000 having only odd digits is

(a) 155 (b) 177 (c) 55 (d) 205
(e) 85

24. Five points are marked on a circle. The number of distinct polygons of three or more sides can be drawn using some (or all) of the five points as vertices is

(a) 10 (b) 12 (c) 14 (d) 16
(e) 18

25. The middle term in the expansion of $\left(1 + \frac{1}{5}\right)^{20}$ is

(a) $\left(\frac{1}{5}\right)^{10}$ (b) $\left(\frac{1}{5}\right)^{11}$
(c) ${}^{20}C_{11}\left(\frac{1}{5}\right)^{11}$ (d) ${}^{20}C_9\left(\frac{1}{5}\right)^9$
(e) ${}^{20}C_{10}\left(\frac{1}{5}\right)^{10}$

26. ${}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + {}^{11}C_3 + {}^{11}C_4 + {}^{11}C_5 =$

(a) 2^6 (b) 2^8 (c) 2^{10} (d) 2^{11}
(e) 2^9

27. If ${}^nP_r = 840$ and ${}^nC_r = 35$, then the value of r is equal to

(a) 2 (b) 4 (c) 6 (d) 3
(e) 5

28. The sum of the coefficients in the expansion of $(1 + 2x - x^2)^{20}$ is

(a) 2^{20} (b) 2^{21} (c) 2^{19} (d) 2^{40}
(e) 2

29. The number of ways a committee of 4 people can be chosen from a panel of 10 people is
(a) 315 (b) 240 (c) 210 (d) 720
(e) 120

30. If $A = \begin{pmatrix} 6 & 2 \\ 7 & -5 \end{pmatrix}$ and $A - B = \begin{pmatrix} -2 & 1 \\ 4 & -9 \end{pmatrix}$, then $B =$
(a) $\begin{pmatrix} -8 & -1 \\ 3 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 8 & 1 \\ -3 & -4 \end{pmatrix}$
(c) $\begin{pmatrix} 4 & 3 \\ 11 & -14 \end{pmatrix}$ (d) $\begin{pmatrix} 8 & 1 \\ 3 & 4 \end{pmatrix}$
(e) $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$

31. The value of the determinant $\begin{vmatrix} bc & ca & ab \\ a^3 & b^3 & c^3 \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \end{vmatrix}$ is
(a) $a^5 - 1$ (b) $a^2bc + ab^2c + abc^2$
(c) $ab(a + b + c)$ (d) $a^4b^4c^4(a + b + c)$
(e) 0

32. If the matrix $\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & k \\ -4 & 2 & 6 \end{bmatrix}$ is singular, then the value of k is equal to
(a) 3 (b) 4
(c) 5 (d) 6
(e) 7

33. If $\begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 19 \\ \alpha & -27 \\ 0 & 14 \end{bmatrix}$, then the value of α is
(a) 5 (b) 4 (c) 7 (d) – 14
(e) – 5

34. If $A^{-1} = \frac{1}{11}\begin{pmatrix} -3 & 4 \\ 5 & -3 \end{pmatrix}$, then $A =$
(a) $\frac{-1}{11}\begin{pmatrix} 3 & 4 \\ 5 & 3 \end{pmatrix}$ (b) $\frac{1}{11}\begin{pmatrix} 3 & 4 \\ 5 & 3 \end{pmatrix}$
(c) $\begin{pmatrix} 3 & -4 \\ -5 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 4 \\ 5 & 3 \end{pmatrix}$
(e) $\begin{pmatrix} -3 & 4 \\ 5 & -3 \end{pmatrix}$

35. The system of equations
$x + y + 2z = 4$
$3x + 3y + 6z = 17$
$5x - 3y + 2z = 27$
has
(a) no solution
(b) finitely many solutions
(c) infinitely many solutions
(d) unique and trivial solution
(e) unique and non-trivial solution

36. The smallest prime number satisfying the inequality $\frac{2n-3}{3} \geq \frac{n-1}{6} + 1$ is
(a) 2 (b) 3 (c) 5 (d) 7
(e) 11

37. The number of integers satisfying the inequality $|n^2 - 100| < 50$ is
(a) 5 (b) 6 (c) 12 (d) 8
(e) 10

38. The solution set of the rational inequality $\frac{x+9}{x-6} \leq 0$ is
(a) $(-\infty, 9) \cup (6, \infty)$ (b) $(-\infty, 9] \cup (6, \infty)$
(c) $(-\infty, 9] \cup [6, \infty)$ (d) $[-9, 6)$
(e) $(-9, 6]$

39. Which of the following sentences is/are statement(s)?
(i) 10 is less than 5.
(ii) All rational numbers are real numbers.
(iii) Today is a sunny day.
(a) (i), (ii) and (iii) (b) (i) and (ii) only
(c) (i) and (iii) only (d) (ii) and (iii) only
(e) (i) only

40. The value of θ with $0 \leq \theta \leq 90°$ and $\sin^2\theta + 2\cos^2\theta = \frac{7}{4}$ is equal to
(a) 15° (b) 30° (c) 45° (d) 60°
(e) 75°

41. The value of $\sin^2 1° + \sin^2 2° + \sin^2 3° + \ldots + \sin^2 88° + \sin^2 89°$ is equal to
(a) $\frac{45}{2}$ (b) $\frac{49}{2}$ (c) $\frac{89}{2}$ (d) 45
(e) 89

42. The value of $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}$ is equal to

(a) $\frac{5}{8}$ (b) $\frac{3}{4}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{3}{8}$
(e) $\frac{5}{4}$

43. The value of $\sin(45^\circ+\theta)-\cos(45^\circ-\theta)$ is equal to

(a) 1 (b) $\cos\theta$ (c) $\sin\theta$ (d) $2\cos\theta$
(e) 0

44. The value of x in $0\le x\le\pi$ such that $\cos 2x=\cos x$ are

(a) 0 and $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ and $\frac{2\pi}{3}$
(c) 0 and $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$ and $\frac{\pi}{3}$
(e) 0 and $\frac{\pi}{2}$

45. The value of $10\tan(\cot^{-1}3+\cot^{-1}7)$ is equal to

(a) 3 (b) 5 (c) 7 (d) 9
(e) 10

46. If $\tan x+\tan y=\frac{5}{6}$ and $\cot x+\cot y=5$, then $\tan(x+y)$ is

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) 5 (d) 6
(e) 1

47. $\frac{\sin 91^\circ+\sin 1^\circ}{\sin 91^\circ-\sin 1^\circ}=$

(a) $\tan 46^\circ$ (b) $\cot 46^\circ$
(c) $\sin 46^\circ$ (d) $\cos 46^\circ$
(e) 1

48. The value of $\cos\left(\cos^{-1}\frac{1}{5}+2\sin^{-1}\frac{1}{5}\right)$ is equal to

(a) $\frac{4}{5}$ (b) $\frac{-4}{5}$ (c) $\frac{3}{5}$ (d) $\frac{-1}{5}$
(e) $\frac{1}{5}$

49. The equation of the line passing through the point $(-3,7)$ with slope zero is

(a) $x=7$ (b) $y=7$
(c) $x=-3$ (d) $y=-3$
(e) $x=0$

50. The line $y=mx+2$ intersects the parabola $y=ax^2+5x-2$ at $(1,5)$. Then the value of $a+m$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

51. If the points $P(7,5)$, $Q(a,2a)$ and $R(12,30)$ are collinear, then the value of a is equal to

(a) 5 (b) 6 (c) 8 (d) 9
(e) 10

52. If the straight line $4x+6y=5$ and $6x+ky=3$ are parallel, then the value of k is equal to

(a) $\frac{-2}{3}$ (b) 8 (c) 9 (d) 10
(e) $\frac{3}{2}$

53. If $(a,2)$ is the point of intersection of the straight lines $y=2x-4$ and $y=x+c$, then the value of c is equal to

(a) -1 (b) 3 (c) -2 (d) -3
(e) 1

54. The maximum value of $z=7x+5y$ subject to $2x+y\le 100$, $4x+3y\le 240$, $x\ge 0$, $y\ge 0$ is

(a) 350 (b) 380 (c) 400 (d) 410
(e) 420

55. A circle with centre at $(3,6)$ passes through $(-1,1)$. Its equation is

(a) $x^2+y^2-6x-12y+3=0$
(b) $x^2+y^2+6x-10y+3=0$
(c) $x^2+y^2-3x-6y+1=0$
(d) $x^2+y^2+5x+9y+5=0$
(e) $x^2+y^2-6x-12y+4=0$

56. The centre and radius of the circle $x^2+y^2-4x+2y=0$ are

(a) $(2,-1)$ and 5 (b) $(4,2)$ and $\sqrt{20}$
(c) $(2,-1)$ and $\sqrt{5}$ (d) $(-2,1)$ and 5
(e) $(-2,1)$ and $\sqrt{5}$

57. The equation of the circle whose radius is $\sqrt{7}$ and concentric with the circle $x^2+y^2-8x+6y-11=0$ is

(a) $x^2+y^2-8x+6y+7=0$
(b) $x^2+y^2-8x+6y+18=0$

(c) $x^2 + y^2 - 8x + 6y - 4 = 0$
(d) $x^2 + y^2 - 8x + 6y - 18 = 0$
(e) $x^2 + y^2 - 8x + 6y - 7 = 0$

58. The vertex of the parabola $y = x^2 - 2x + 4$ is shifted p units to the right and then q units up. If the resulting point is (4, 5), then the values of p and q respectively are
(a) 2 and 3 (b) 3 and 5
(c) 5 and 2 (d) 3 and 2
(e) 1 and 2

59. The vertex of the parabola $y = (x - 2)(x - 8) + 7$ is
(a) (5, 2) (b) (5, – 2)
(c) (– 5, – 2) (d) (– 5, 2)
(e) (2, 8)

60. The major and minor axis of the ellipse $400x^2 + 100y^2 = 40000$ respectively are
(a) 100 and 20 (b) 20 and 10
(c) 40 and 20 (d) 400 and 100
(e) 16 and 8

61. The eccentricity of the ellipse $x^2 + \frac{y^2}{4} = 1$ is
(a) $\sqrt{3}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{\sqrt{3}}{2}$
(e) $\frac{1}{\sqrt{3}}$

62. The latus rectum of the hyperbola $3x^2 - 2y^2 = 6$ is
(a) $\frac{3}{\sqrt{2}}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) 3
(e) $3\sqrt{2}$

63. If $\mathbf{u} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{v} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$, then $|\mathbf{u} \times \mathbf{v}|^2 + |\mathbf{u} \cdot \mathbf{v}|^2 =$
(a) 640 (b) 630 (c) 690 (d) 740
(e) 730

64. The direction cosines of the vector $\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$ are
(a) $\left(\frac{1}{\sqrt{10}}, \frac{-5}{\sqrt{10}}, \frac{8}{\sqrt{10}}\right)$ (b) $\left(\frac{1}{3\sqrt{10}}, \frac{-5}{3\sqrt{10}}, \frac{8}{3\sqrt{10}}\right)$
(c) $\left(\frac{1}{3}, \frac{-5}{3}, \frac{8}{3}\right)$ (d) $\left(\frac{1}{3\sqrt{10}}, \frac{-1}{3\sqrt{10}}, \frac{1}{3\sqrt{10}}\right)$
(e) $\left(\frac{1}{3\sqrt{10}}, \frac{5}{3\sqrt{10}}, \frac{8}{3\sqrt{10}}\right)$

65. If $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{b} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and θ is the angle between them, then $\tan\theta =$
(a) $\frac{\sqrt{38}}{4}$ (b) $\frac{\sqrt{26}}{4}$ (c) $\frac{\sqrt{26}}{5}$ (d) $\frac{\sqrt{26}}{6}$
(e) $\frac{\sqrt{38}}{6}$

66. The value of λ such that the vectors $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} + 2\lambda\hat{\mathbf{j}}$ are perpendicular is
(a) 0 (b) 1 (c) 2 (d) 3
(e) 4

67. The values of α so that $|\alpha\hat{\mathbf{i}} + (\alpha + 1)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}| = 3$, are
(a) 2, – 4 (b) 1, 2 (c) – 1, 2 (d) – 2, 4
(e) 1, – 2

68. If $\mathbf{a} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and $\mathbf{b} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, then the value of $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})$ is equal to
(a) 8 (b) 7 (c) 9 (d) 11
(e) 13

69. Let $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\mathbf{b} = \lambda\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$. If the projection of $\mathbf{a}$ on $\mathbf{b}$ is equal to the projection of $\mathbf{b}$ on $\mathbf{a}$, then the values of λ are
(a) $\pm\sqrt{7}$ (b) $\pm\sqrt{3}$ (c) ± 5 (d) ± 3
(e) $\pm\sqrt{5}$

70. If $|\mathbf{a}| = 2$, $|\mathbf{b}| = 3$ and $\mathbf{a} \cdot \mathbf{b} = 4$, $|\mathbf{a} - \mathbf{b}|$ is equal to
(a) $\sqrt{5}$ (b) $\sqrt{7}$ (c) $\sqrt{6}$ (d) 5
(e) 6

71. Which one of the following points lies on the straight line $\frac{x-1}{2} = \frac{y+1}{4} = \frac{z-2}{-2}$?
(a) (2, 6, – 2) (b) (4, 3, 1)
(c) (3, 4, – 1) (d) (3, 3, 0)
(e) (6, 2, – 1)

72. A plane passes through the point (0, 1, 1) and has normal vector $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$. Its equation is
(a) $x + y + z = 1$ (b) $x + y + z = 2$
(c) $2x + 2y + 2z = 1$ (d) $y + z = 2$
(e) $y + z = 1$

73. The distance of the point (4, 2, 3) from the plane $\mathbf{r} \cdot (6\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 9\hat{\mathbf{k}}) = 46$ is
(a) $\frac{23}{5}$ (b) $\frac{46}{11}$ (c) $\frac{45}{11}$ (d) $\frac{11}{45}$
(e) $\frac{5}{23}$

74. The sum of the intercepts made by the plane $\mathbf{r}\cdot(3\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})=18$ on the co-ordinate axes is

(a) 30 (b) 18
(c) 33 (d) 36
(e) 27

75. The point at which the line $\frac{x-2}{1}=\frac{y-4}{-5}=\frac{z+3}{4}$ intersects the xy-plane is

(a) $\left(\frac{11}{4},\frac{1}{4},0\right)$ (b) $\left(\frac{5}{4},\frac{1}{4},0\right)$
(c) $\left(\frac{11}{4},\frac{3}{4},0\right)$ (d) $\left(\frac{7}{4},\frac{1}{4},0\right)$
(e) $\left(\frac{11}{4},\frac{7}{4},0\right)$

76. The Cartesian equation of the line passing through the points $(1,-1,2)$ and $(7,0,5)$ is

(a) $\frac{x-1}{4}=\frac{y+1}{1}=\frac{z-2}{2}$
(b) $\frac{x-7}{1}=\frac{y}{-1}=\frac{z-5}{2}$
(c) $\frac{x-1}{7}=\frac{y+1}{1}=\frac{z-2}{5}$
(d) $\frac{x-1}{6}=\frac{y+1}{1}=\frac{z-2}{3}$
(e) $\frac{x-7}{6}=\frac{y}{-1}=\frac{z-5}{3}$

77. The angle between the planes $x+y+z=1$ and $x-2y+3z=1$ is

(a) $\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$ (b) $\cos^{-1}\left(\frac{5}{\sqrt{42}}\right)$
(c) $\cos^{-1}\left(\frac{3}{\sqrt{42}}\right)$ (d) $\cos^{-1}\left(\frac{1}{\sqrt{42}}\right)$
(e) $\cos^{-1}\left(\frac{4}{\sqrt{42}}\right)$

78. The equation of the plane passing through the intersection of the planes $x+2y-z=3$ and $x+y-3z=5$ and passing through the point $(1,-1,0)$ is

(a) $x+7y+6z+6=0$ (b) $x-6y-7z+5=0$
(c) $x+7y+6z+5=0$ (d) $x+6y-7z-5=0$
(e) $x+6y+7z+5=0$

79. The average marks of 30 students in a class was 80. After two students left out of the class, the average marks of the remaining students was 82. Then the average marks of the two left out students is

(a) 62 (b) 72 (c) 70 (d) 52
(e) 60

80. Two dice are rolled. If each die has six faces which are numbered 2, 3, 5, 7, 11, 13, then the probability that sum of the numbers on the top faces being a prime number is

(a) $\frac{1}{6}$ (b) $\frac{5}{36}$ (c) $\frac{1}{18}$ (d) $\frac{1}{9}$
(e) $\frac{1}{12}$

81. Three different numbers are chosen at random from the set {1, 2, 3, 4, 5} and arranged in increasing order. The probability that the resulting sequence is an AP is

(a) $\frac{1}{2}$ (b) $\frac{3}{10}$ (c) $\frac{1}{5}$ (d) $\frac{1}{10}$
(e) $\frac{2}{5}$

82. In an examination, 20% of the students scored 70 marks, 40% scored 80 marks, 30% scored 90 marks and the rest scored 100 marks. Then the mean score of the students is

(a) 82 (b) 85 (c) 83 (d) 90
(e) 93

83. If A and B are mutually exclusive events such that $p(A)=0.5$ and $p(A\cup B)=0.75$, then $P(B)$ is equal to

(a) 0.4 (b) 0.25
(c) 0.5 (d) 0.6
(e) 0.75

84. A jar contains 7 black balls, 6 yellow balls, 4 green balls and 3 red balls. All of them are of same size and weight. If a ball is drawn at random, then the probability of the ball being red is

(a) $\frac{1}{5}$ (b) $\frac{3}{20}$ (c) $\frac{1}{10}$ (d) $\frac{3}{10}$
(e) $\frac{1}{20}$

85. Let the probability distribution of a random variable X be given by

X	-1	0	1	2	3
$p(X)$	a	$2a$	$3a$	$4a$	$5a$

Then the expectation of X is

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{15}$

(e) $\frac{5}{3}$

86. Let $f(x)=\begin{cases}1-5x, & \text{if } x<-2\\ x^2-2x, & \text{if } -2\le x\le 1.\\ -1+2x, & \text{if } x>1\end{cases}$

Then the value of $f(-1)$ is equal to

(a) -3 (b) 3 (c) -1 (d) 1

(e) 0

87. The general solution of $\frac{dy}{dx}=\frac{2x-y}{x+2y}$ is given by

(a) $x^2-y^2-xy=C$

(b) $x^2+y^2+xy=C$

(c) $x^2+2y^2+y+x=C$

(d) $2x^2+y^2+xy+y=C$

(e) $x^2-y^2-xy+x=C$

88. $\lim_{x\to 3}\frac{e^{x-3}-x+1}{x^2-\log(x-2)}$ is equal to

(a) $\frac{-1}{3}$ (b) $\frac{-2}{9}$ (c) $\frac{-1}{2}$ (d) $\frac{-1}{4}$

(e) $\frac{-1}{9}$

89. $\lim_{x\to 4}\frac{\sqrt{x^2+9}-5}{x-4}$ is equal to

(a) $\frac{2}{5}$ (b) $\frac{8}{25}$ (c) 0 (d) $\frac{8}{5}$

(e) $\frac{4}{5}$

90. Let $f(x)=\begin{cases}cx^2+2x, & \text{if } x<2\\ 2x+4, & \text{if } x\ge 2\end{cases}$

If the function f is continuous on $(-\infty,\infty)$, then the value of c is equal to

(a) 4 (b) 2 (c) 3 (d) 1

(e) 5

91. $\lim_{x\to 0}\frac{x^{100}\sin 7x}{(\sin x)^{101}}$ is equal to

(a) 7 (b) $\frac{1}{7}$ (c) 14 (d) 1

(e) 0

92. Let $f(x)=\frac{5}{2}x^2-e^x$. Then the value of c such that $f''(c)=0$ is

(a) 1 (b) $\log 5$

(c) $5e$ (d) e^5

(e) 0

93. If $y=(\cos x)^{2x}$, then $\frac{dy}{dx}$ is equal to

(a) $2(\cos x)^{2x}(\sin x-x\tan x)$

(b) $2(\cos x)^{2x}[\log(\cos x)+x\tan x]$

(c) $2(\sin x)^{2x}[\log(\cos x)-x\tan x]$

(d) $2(\sin x)^{2x}x\cot x$

(e) $2(\cos x)^{2x}[\log(\cos x)-x\tan x]$

94. If $x^3+2xy+\frac{1}{3}y^3=\frac{11}{3}$, then $\frac{dy}{dx}$ at $(2,-1)$ is

(a) -2 (b) 2 (c) 5 (d) -5

(e) -10

95. Let $f(x)=\begin{cases}x^2, & \text{for } x\le 1\\ 1, & \text{for } 1<x\le 3\\ 5-2x, & \text{for } x>3\end{cases}$

Then $f'(6)$ is equal to

(a) -7 (b) 3 (c) -2 (d) -3

(e) 2

96. Given $F(x)=(f(g(x)))^2$, $g(1)=2$, $g'(1)=3$, $f(2)=4$ and $f'(2)=5$. Then the value of $F'(1)$ is equal to

(a) 25 (b) 100

(c) 75 (d) 50

(e) 120

97. If $y=2+\sqrt{u}$ and $u=x^3+1$, then $\frac{dy}{dx}=$

(a) $\frac{x^2}{2\sqrt{x^3+1}}$ (b) $\frac{3x^2}{\sqrt{x^3+1}}$

(c) $\frac{3x^2}{2\sqrt{x^3+1}}$ (d) $3x^2\sqrt{x^3+1}$

(e) $x^2\sqrt{x^3+1}$

98. The equation of the tangent to $y=-2x^2+3$ at $x=1$ is

(a) $y=-4x$ (b) $y=-4x+5$

(c) $y=4x$ (d) $y=4x+5$

(e) $y=-4x+3$

99. The function f given by $f(x) = x^3 e^x$ is increasing on the interval

(a) $(0, \infty)$ (b) $(3, \infty)$
(c) $(-3, \infty)$ (d) $(-3, 3)$
(e) $(-\infty, -3)$

100. Let $f(x) = \sqrt{x}$, $4 \le x \le 16$. If the point $c \in (4, 16)$ is such that the tangent line to the graph of f at $x = c$ is parallel to the chord joining (16, 4) and (4, 2), then the value of c is

(a) 7 (b) 9 (c) 10 (d) 11
(e) 14

101. The function f given by $f(x) = (x^2 - 3)e^x$ is decreasing on the interval

(a) $(-3, \infty)$ (b) $(1, \infty)$
(c) $(-\infty, 1)$ (d) $(-\infty, -3)$
(e) $(-3, 1)$

102. The equation of normal to the curve $y = \dfrac{2}{x^2}$ at the point on the curve where $x = 1$, is

(a) $4y - x - 7 = 0$ (b) $y - 4x + 2 = 0$
(c) $4y + x - 9 = 0$ (d) $y - x - 1 = 0$
(e) $4y + x + 7 = 0$

103. The local minimum value of the function f given by $f(x) = x^2 - x$, $x \in \mathbf{R}$, is

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$
(c) $\dfrac{-1}{4}$ (d) $\dfrac{3}{4}$
(e) $\dfrac{-1}{2}$

104. $\int 3x^2(x^3 + 1)^{10} dx =$

(a) $\dfrac{(x^3+1)^{11}}{11} + C$ (b) $\dfrac{(x^3+1)^9}{9} + C$
(c) $\dfrac{(x^3+1)^{11}}{33} + C$ (d) $\dfrac{(x^3+1)^{11}}{11} + x^3 + C$
(e) $\dfrac{(x^3+1)^{11}}{10} + C$

105. $\int \dfrac{2x + \sin 2x}{1 + \cos 2x} dx =$

(a) $x^2 \sec x + C$ (b) $x + \tan x + C$
(c) $x^2 \tan x + C$ (d) $x \sec x + C$
(e) $x \tan x + C$

106. $\int \dfrac{1}{x^2 - 25} dx =$

(a) $\log\left|\dfrac{x-5}{x+5}\right| + C$ (b) $\log\left|\dfrac{x+5}{x-5}\right| + C$
(c) $\dfrac{1}{5}\log\left|\dfrac{x-5}{x+5}\right| + C$ (d) $\dfrac{1}{10}\log\left|\dfrac{x-5}{x+5}\right| + C$
(e) $\dfrac{1}{5}\log\left|\dfrac{x+5}{x-5}\right| + C$

107. $\int \dfrac{1}{x(\log x)} dx =$

(a) $\log|\log x| + C$ (b) $\dfrac{(\log|x|)^2}{2} + C$
(c) $\log|x| + C$ (d) $\dfrac{1}{\log|x|} + C$
(e) $\dfrac{1}{(\log|x|)^2} + C$

108. $\int e^x \sec x(1 + \tan x)\, dx =$

(a) $e^x \tan x + C$ (b) $e^x + \sec x + C$
(c) $e^{-x} \sec x + C$ (d) $e^x + \tan x + C$
(e) $e^x \sec x + C$

109. $\int \dfrac{1}{x + \sqrt{x}} dx =$

(a) $\log|1 + \sqrt{x}| + C$ (b) $2\log|1 - \sqrt{x}| + C$
(c) $\log|1 - \sqrt{x}| + C$ (d) $2\log|1 + \sqrt{x}| + C$
(e) $2\log|x + \sqrt{x}| + C$

110. $\int \sec^2(5x - 1)\, dx =$

(a) $\dfrac{1}{5}\tan(5x - 1) + C$ (b) $5\tan(5x - 1) + C$
(c) $\tan(5x - 1) + C$ (d) $\cot(5x - 1) + C$
(e) $\dfrac{1}{5}\cot(5x - 1) + C$

111. $\int_0^{\frac{\pi}{2}} \dfrac{1}{1 + \cot^4 x} dx =$

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{4}$ (c) π (d) $\dfrac{\pi}{8}$
(e) 2π

112. The value of $\int_{-10}^{10} (0.0002x^3 - 0.3x + 20)dx$ is equal to

(a) 423 (b) 400 (c) 378 (d) 410
(e) 390

113. The area enclosed by the curve $x = 3\cos\theta$, $y = 2\sin\theta$, $0 \le \theta \le \pi$, is (in square units)

(a) 9π (b) 6π (c) 4π (d) 3π
(e) 2π

114. The area of the region bounded by $y = |x|$, $y = 0$, $x = 3$ and $x = -3$ is (in square units)

(a) 3 (b) 6 (c) 7 (d) 9
(e) 10

115. The value of $\int_e^{e^2} \frac{1}{x}\, dx$ is equal to

(a) e (b) 1 (c) e^2 (d) $e^2 - e$
(e) 0

116. $\int_{-3}^{3} |x + 2|\, dx =$

(a) 17 (b) 9 (c) 14 (d) 13
(e) 12

117. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \sqrt{x^2 + \left(\frac{dy}{dx}\right)^{3/2}} = 0$ are respectively

(a) 2, 4 (b) 2, 3 (c) 2, 2 (d) 3, 4
(e) 4, 3

118. The general solution of the differential equation $xy' + y = x^2$, $x > 0$ is

(a) $y = \frac{x^2}{2} + Cx$ (b) $y = \frac{x^3}{3} + C$
(c) $y = \frac{x^2}{3} + C$ (d) $y = \frac{x^3}{3} + \frac{C}{x}$
(e) $y = \frac{x^2}{3} + \frac{C}{x}$

119. The integrating factor of the differential equation $3xy' - y = 1 + \log x$, $x > 0$ is

(a) $\log x$ (b) $\frac{1}{x}$
(c) $x^{-1/3}$ (d) $\frac{1}{x^3}$
(e) $x^{1/3}$

120. Elimination of arbitrary constants A and B from $y = \frac{A}{x} + B$, $x > 0$ leads to the differential equation

(a) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$ (b) $x^2\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$
(c) $x^2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ (d) $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$
(e) $x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

Answers

Physics & Chemistry

1.	(a)	2.	(e)	3.	(b)	4.	(d)	5.	(a)	6.	(a)	7.	(e)	8.	(a)	9.	(b)	10.	(d)
11.	(e)	12.	(d)	13.	(c)	14.	(e)	15.	(d)	16.	(a)	17.	(b)	18.	(d)	19.	(a)	20.	(c)
21.	(b)	22.	(c)	23.	(a)	24.	(b)	25.	(e)	26.	(e)	27.	(e)	28.	(a)	29.	(d)	30.	(d)
31.	(a)	32.	(c)	33.	(b)	34.	(a)	35.	(e)	36.	(a)	37.	(b)	38.	(a)	39.	(a)	40.	(e)
41.	(e)	42.	(a)	43.	(b)	44.	(d)	45.	(a)	46.	(c)	47.	(b)	48.	(a)	49.	(d)	50.	(c)
51.	(c)	52.	(a)	53.	(e)	54.	(b)	55.	(d)	56.	(c)	57.	(e)	58.	(d)	59.	(d)	60.	(b)
61.	(e)	62.	(c)	63.	(b)	64.	(c)	65.	(a)	66.	(b)	67.	(*)	68.	(b)	69.	(c)	70.	(c)
71.	(c)	72.	(e)	73.	(d)	74.	(a)	75.	(b)	76.	(d)	77.	(c)	78.	(e)	79.	(c)	80.	(e)
81.	(b)	82.	(b)	83.	(a)	84.	(b)	85.	(e)	86.	(d)	87.	(a)	88.	(b)	89.	(e)	90.	(c)
91.	(e)	92.	(d)	93.	(e)	94.	(d)	95.	(c)	96.	(a)	97.	(c)	98.	(d)	99.	(e)	100.	(c)
101.	(d)	102.	(a)	103.	(e)	104.	(c)	105.	(d)	106.	(a)	107.	(b)	108.	(e)	109.	(c)	110.	(a)
111.	(c)	112.	(b)	113.	(c)	114.	(d)	115.	(b)	116.	(c)	117.	(b)	118.	(d)	119.	(a)	120.	(b)

Mathematics

1.	(c)	2.	(d)	3.	(a)	4.	(c)	5.	(b)	6.	(c)	7.	(b)	8.	(c)	9.	(a)	10.	(a)
11.	(d)	12.	(c)	13.	(d)	14.	(c)	15.	(d)	16.	(e)	17.	(c)	18.	(e)	19.	(a)	20.	(d)
21.	(d)	22.	(c)	23.	(a)	24.	(d)	25.	(e)	26.	(c)	27.	(b)	28.	(a)	29.	(c)	30.	(d)
31.	(e)	32.	(c)	33.	(b)	34.	(d)	35.	(a)	36.	(c)	37.	(e)	38.	(d)	39.	(b)	40.	(b)
41.	(c)	42.	(b)	43.	(e)	44.	(a)	45.	(b)	46.	(e)	47.	(a)	48.	(d)	49.	(b)	50.	(e)
51.	(e)	52.	(c)	53.	(a)	54.	(d)	55.	(e)	56.	(c)	57.	(b)	58.	(d)	59.	(b)	60.	(c)
61.	(d)	62.	(e)	63.	(b)	64.	(b)	65.	(b)	66.	(d)	67.	(e)	68.	(d)	69.	(e)	70.	(a)
71.	(d)	72.	(b)	73.	(c)	74.	(c)	75.	(a)	76.	(d)	77.	(a)	78.	(e)	79.	(d)	80.	(a)
81.	(e)	82.	(c)	83.	(b)	84.	(b)	85.	(e)	86.	(b)	87.	(a)	88.	(e)	89.	(e)	90.	(d)
91.	(a)	92.	(b)	93.	(e)	94.	(a)	95.	(c)	96.	(e)	97.	(c)	98.	(b)	99.	(c)	100.	(b)
101.	(e)	102.	(a)	103.	(c)	104.	(a)	105.	(e)	106.	(d)	107.	(a)	108.	(e)	109.	(d)	110.	(a)
111.	(b)	112.	(b)	113.	(b)	114.	(d)	115.	(b)	116.	(d)	117.	(a)	118.	(e)	119.	(c)	120.	(a)

Answer *with* Explanations

Physics

1. **(a)** Given that,

Angular velocity of disc, $\omega_0 = 27.5 \text{ rads}^{-1}$

Angular acceleration of disc, $\alpha = -10 \text{ rads}^{-2}$

Using equation of rotational kinematics,

$$\omega = \omega_0 + \alpha t$$

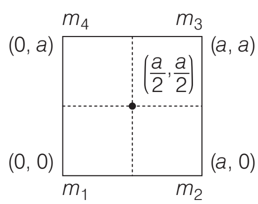
Substituting values, we get

$$0 = 27.5 + (-10)t$$

$\Rightarrow \quad 27.5 = 10 \times t$ ($\because$ at final position disc is at rest)

$\Rightarrow \quad t = 2.75 \text{ s}$

2. **(e)** Consider the figure of square shown below,



Let, the side of the square be a.

The co-ordinate of centre of mass about X-axis is given by

$$x_{cm} = \frac{m_1 \times a + m_2 \times a + m_3 \times a + m_4 \times a}{m_1 + m_2 + m_3 + m_4} = \frac{a}{2}$$

Substituting given values, we get

$$x_{cm} = \frac{1 \times 0 + 2 \times a + 1 \times a + m_4 0}{1 + 2 + 1 + m_4} = \frac{a}{2}$$

$$\Rightarrow \quad \frac{2a + a}{4 + m_4} = \frac{a}{2}$$

$$\Rightarrow \quad 6a = 4a + m_4 a \quad \Rightarrow \quad m_4 = 2 \text{ kg}$$

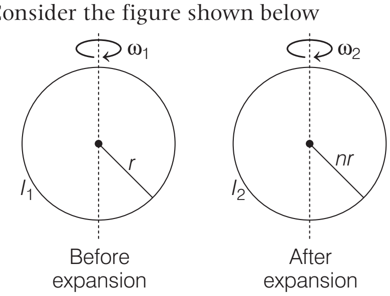
Similarly,

$$y_{cm} = \frac{m_1 \times 0 + m_2 \times 0 + m_3 \times a + m_4 \times a}{m_1 + m_2 + m_3 + m_4} = \frac{a}{2}$$

$$\Rightarrow \quad \frac{a + m_4 a}{4 + m_4} = \frac{a}{2}$$

$$\Rightarrow \quad m_4 = 2 \text{ kg}$$

3. **(b)** Consider the figure shown below



As, there is no external torque acting on the system, so the total angular momentum remains conserved,

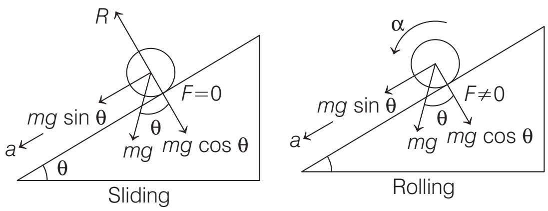
$$L_i = L_f$$

$$I_1\omega_1 = I_2\omega_2$$

$$\left(\frac{2}{5}mr^2\right)\omega = \left(\frac{2}{5}m(nr)^2\right)\omega_2$$

$$\Rightarrow \qquad \omega = n^2\omega_2 \quad \text{or} \quad \omega_2 = \frac{\omega}{n^2}$$

4. *(d)* Consider the figure shown below, for both motion,



When the ring is sliding on inclined plane

$$a = -g\sin\theta$$

As, at final position the ring is at rest ($v = 0$), using kinematic relation, we get

$$v = u + at_2$$

$$0 = u + at_2$$

$$\Rightarrow \qquad u = g\sin\theta t_2$$

If L be the length of inclined plane, then

$$\frac{L}{t_2} = g\sin\theta t_2 \quad \Rightarrow \quad t_2^2 = \frac{L}{g\sin\theta} \qquad \dots \text{(i)}$$

When the ring rolls down the plane,

$$a = -\frac{g\sin\theta}{\left(1 + \frac{I}{mR^2}\right)}$$

$$a = -\frac{g\sin\theta}{2} \qquad (\because I = mR^2)$$

Similarly,
$$v = u + at_1$$

$$u = -\frac{g\sin\theta}{2}t_1$$

$$\frac{L}{t_1} = \frac{g\sin\theta t_1}{2}$$

$$\Rightarrow \qquad t_1^2 = \frac{2L}{g\sin\theta} \qquad \dots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \qquad \frac{t_2^2}{t_1^2} = \frac{L/g\sin\theta}{2L/g\sin\theta} = \frac{1}{2}$$

5. *(a)* Given that,

Distance between sun and earth is d.

For a circular orbit, using Kepler's rule of, angular momentum is given by

$$L = mvd = m\sqrt{\frac{Gm}{d}} \cdot d$$

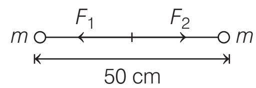
$$\left(\because \text{From Kepler's third law, } v^2 = \frac{Gm}{r}\right)$$

$$\Rightarrow \qquad L \propto \sqrt{d}$$

6. *(a)* Given that,

Mass of each object, $m = 50$ kg

Distance of separation, $d = 50$ cm



Let m_1 be the mass at centre.

The net gravitational force at mid-point becomes,

$$F_G = F_1 - F_2 = \frac{G \times 50 \times m_1}{(25)^2} - \frac{G \times m_1 \times 50}{(25)^2} = 0$$

7. *(e)* The acceleration due to gravity at height h is given by,

$$g'_1 = \frac{GM}{(R+h)^2} = \frac{gR^2}{(R+h)^2} \qquad (\because GM = gR^2)$$

$$g'_1 = g\left(\frac{R}{R+h}\right)^2 \simeq g\left(1 - \frac{2h}{R}\right)$$

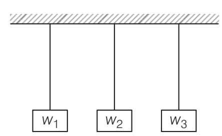
The acceleration due to gravity at depth d is,

$$g'_2 = g\left(1 - \frac{d}{R}\right)$$

The ratio of weight at $h = R/10$ and $d = R/10$ becomes

$$\frac{w_1}{w_2} = \frac{mg'_1}{mg'_2} = \frac{\left(1 - \frac{2h}{R}\right)}{\left(1 - \frac{d}{R}\right)} = \frac{\left(1 - \frac{2(R/10)}{R}\right)}{1 - \frac{\left(\frac{R}{10}\right)}{R}} = \frac{8}{9}$$

8. *(a)* Consider the figure shown below,



The Young's modulus is given by

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$Y = \frac{FL}{Al}$$

$$Y = \frac{mgL}{Al} = \frac{wL}{Al}$$

For first wire 1, $Y_1 = \frac{w_1 L}{A_1 l}$

$$4 = \frac{w_1 L}{1 \times l} \Rightarrow w_1 = \frac{4l}{L} \quad \ldots \text{(i)}$$

For second wire 2, $Y_2 = \frac{w_2 L}{A_2 l}$

$$2 = \frac{w_2 L}{2 \times l}$$

$$w_2 = \frac{4l}{L} \quad \ldots \text{(ii)}$$

For third wire,

$$Y_3 = \frac{w_3 L}{A_3 l}$$

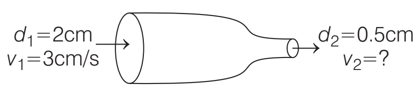
$$1 = \frac{w_3 L}{4l}$$

$$w_3 = \frac{4l}{L} \quad \ldots \text{(iii)}$$

The ratio of weights is given by

$$w_1 : w_2 : w_3 = \frac{4l}{L} : \frac{4l}{L} : \frac{4l}{L} = 1:1:1$$

9. *(b)* Consider the figure shown below



Let the density of fluid flowing through pipe is constant.

By equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow \left(\frac{\pi}{4} d_1^2\right) v_1 = \left(\frac{\pi}{4} d_2^2\right) v_2$$

$$\Rightarrow \frac{\pi}{4} (2\text{ cm})^2 \times 3\text{cm/s} = \frac{\pi}{4} (0.5\text{ cm})^2 \times v_2$$

$$\Rightarrow v_2 = 48\text{ cms}^{-1}$$

10. *(d)* Given that,

Density of kerosene $(\rho_K) = 800\text{ kg/m}^3$

The relative density is given by

$$\text{Relative density} = \frac{\text{Density of kerosene}}{\text{Density of water}}$$

$$= \frac{800\text{ kg/m}^3}{1000\text{ kg/m}^3} = 0.8$$

11. *(e)* Let the radius of the solid sphere of volume V be r.

The viscous force using Stoke's law is

$$F_v = 6\pi\eta r v$$

where, η is the coefficient of viscosity of medium.

For another solid sphere of volume $27V$,

$$V' = \frac{4}{3}\pi r_1^3$$

$$27V = \frac{4}{3}\pi r_1^3$$

$$r_1 = \left(\frac{3 \times 27V}{4\pi}\right)^{1/3}$$

$$\left(\because V = \frac{4}{3}\pi r^3 \Rightarrow r^3 = \frac{3V}{4\pi}\right)$$

$$r_1 = (27)^{1/3} r$$

$$r_1 = 3r$$

The viscous force for another sphere becomes,

$$(F_v)_1 = 6\pi\,\eta\, r_1 v$$

$$= 6\pi\,\eta\,(3r) v = 3F_v$$

$$\Rightarrow (F_v)_1 = 3F$$

12. *(d)* By principle of calorimetry,

Heat gain = Heat lost

$m_1 c(T_1 - T) = m_2 c(T - T_2)$

where, m_1 is the rate of water flow from tap 1, m_2 is the rate of water flow from tap 2, T_1 is temperature of water from tap 1 and T_2 is temperature of water from tap 2.

$$2c(20 - T) = c(T - 80)$$

$$\Rightarrow 40 - 2T = T - 80$$

$$\Rightarrow 40 + 80 = 3T$$

$$T = \frac{120}{3} = 40^\circ\text{C}$$

13. *(c)* The ideal gas equation for an adiabatic process is

$$pV^\gamma = \text{constant}$$

$$\Rightarrow p_1 V_1^\gamma = p_2 V_2^\gamma$$

For monoatomic gas, $\gamma = \frac{5}{3}$

$$\Rightarrow p_1 V_1^{5/3} = p_2 \left(\frac{V_1}{27}\right)^{5/3} \quad \left[\text{given } V_2 = \frac{1}{27} V_1\right]$$

$$\Rightarrow p_1 V_1^{5/3} = p_2 \left(\frac{V_1^{5/3}}{3^{3(5/3)}}\right)$$

$$\Rightarrow \quad p_1 V_1^{5/3} = p_2\left(\frac{V_1^{5/3}}{3^5}\right)$$

$$\Rightarrow \quad p_1 (3)^5 = p_2$$

$$\Rightarrow \quad p_2 = 243 p_1$$

Hence, option (c) is correct.

14. *(e)* For diatomic gas, degree of freedom, $f = 5$

∴ Specific heat at constant volume,

$$C_V = \frac{f}{2}R = \frac{5}{2}R$$

and specific heat at constant pressure,

$$C_p = C_V + R = \frac{5}{2}R + R = \frac{7}{2}R$$

Hence, option (e) is correct.

15. *(d)* The pressure using ideal gas equation is

$$pV = nRT$$

$$p = \frac{nRT}{V}$$

where, n is number of mole, R is gas constant, T is gas temperature and V is volume of cubical box.

At $T_1 = 27°$ C

$$p_1 = \frac{3 \times 8.315 \times (27 + 273)}{(0.170)^3} \qquad (\because V = a^3)$$

$$= 1523203.75 \text{ Pa}$$

The force exerted by gas on cubical box is

$$F_1 = p_1 \times A$$

$$= 1523203.75 \times A \qquad \ldots \text{(i)}$$

Similarly,

At $T_2 = 127°$C

$$p_2 = \frac{3 \times 8.315 \times (127 + 273)}{(0.170)^3}$$

$$= 2030938 \cdot 33 \text{ Pa}$$

The force becomes,

$$F_2 = P_2 \times A$$

$$= 2030938.33 \times A \qquad \ldots \text{(ii)}$$

The ratio of forces become,

$$\frac{F_1}{F_2} = \frac{1523203.75 \times A}{2030938.33 \times A} = 0.75$$

$$\frac{F_1}{F_2} = \frac{3}{4}$$

16. *(a)* The monoatomic gas has 3 degree of freedom, one each along for each axis.

The kinetic energy per molecule for a monoatomic gas is

$$E = \frac{3}{2}kT = \frac{3}{2}(1.3 \times 10^{-23}) \times (27 + 273)$$

$$= 5.85 \times 10^{-21} \text{ J}$$

Hence, option (a) is correct.

17. *(b)* The travelling wave is given by

$$y = a\sin(\omega t - kx)$$

$$v = \frac{dy}{dt} = \omega a \cos(\omega t - kx)$$

$$a = \frac{dv}{dt} = -\omega^2 a \sin(\omega t - kx)$$

For maximum acceleration, $\sin(\omega t - kx) = 1$

$$|a_{\text{max}}| = \omega^2 a$$

Hence, option (b) is correct.

18. *(d)* The resultant amplitude of particle in simple harmonic motion is given by

$$A_{\text{resultant}} = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}$$

It is given

$$A_{\text{resultant}} = A_1 = A_2 = A$$

$$\Rightarrow \quad A^2 = 2A^2 + 2A^2 \cos\phi$$

$$-A^2 = 2A^2 \cos\phi$$

$$\cos\phi = -\frac{1}{2}$$

$$\phi = \frac{2\pi}{3}$$

19. *(a)* Frequency of wave is given by

$$f_1 = \frac{nv}{2L} \qquad \ldots \text{(i)}$$

For $(n+1)$th harmonic frequency,

$$f_2 = \frac{(n+1)v}{2L} \qquad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$f_2 - f_1 = \frac{(n+1)v}{2L} - \frac{nv}{2L}$$

$$f_2 - f_1 = \frac{v}{2L}$$

$$240 - 200 = \frac{v}{2L}$$

$$40 = \frac{343}{2 \times L} \quad (\because \text{speed of sound, } v = 343 \text{m/s})$$

$$L = 4.28 \text{ m}$$

The fundamental frequency $(n = 1)$ is

$$f = \frac{v}{2L} = \frac{343}{2 \times 4.28}$$

$$= 40.07 \approx 40 \text{ Hz}$$

20. *(c)* The fundamental frequency produced in a string is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{T}{m}} = \frac{1}{2\pi}\sqrt{\frac{T}{A\rho}}$$

Area of wire, $A = 2\pi rl$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{T}{2\pi rl\rho}}$$

As, length, tension and material of both strings are same.

$$\Rightarrow \quad f \propto \frac{1}{\sqrt{r}}$$

$$\therefore \quad \frac{f_1}{f_2} = \sqrt{\frac{r_2}{r_1}}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{f_2^2}{f_1^2} = \left(\frac{900}{1600}\right) = \frac{81}{256}$$

$$\therefore \quad d_1 : d_2 = 2r_1 : 2r_2 = 81 : 256$$

21. *(b)* Given that,

Velocity of an object, $v = 100\ \text{m/s}$

Frequency of note $f_s = 400\ \text{Hz}$

When a source is moving and observer is stationary then by using Doppler effect the frequency heard by observer,

$$f_0 = f_s\left(\frac{c}{c+v}\right)$$

where, c = speed of sound in air = 300 m/s.

$$f_0 = 400\left(\frac{300}{300+100}\right)$$

$$= 400 \times \frac{300}{400} = 300\ \text{Hz}$$

22. *(c)* According to Gauss's law in electrostatic, the net flux through any closed surface, in an electric field is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface.

i.e., $$\phi_{\text{net}} = \frac{Q}{\varepsilon_0}$$

Hence, electric flux through a face of cube is

$$\phi_f = \frac{1}{6}\frac{Q}{\varepsilon_0} \qquad (\because \text{cube has 6 faces})$$

23. *(a)* Given, $C = 10\ \mu\text{F}$

When a dielectric plate is introduced between plates of capacitor, then capacitance become

$$C_d = \frac{K\varepsilon_0 A}{d} = KC$$

Here, $C_d = 2C \Rightarrow 2C = KC$

or $K = 2$

24. *(b)* Given, electric potential, $V = 4z^2$ V

As we know, electric field, $E = \frac{-dV}{dr}$

$$\Rightarrow \quad E = -\left(\frac{dV}{dx} + \frac{dV}{dy} + \frac{dV}{dz}\right)$$

$$= -\left[0 + 0 + \frac{d}{dz}(4z^2)\right] = -8z\ \text{Vm}^{-1}$$

At, $r(1\text{m}, 0, 2\text{m})$, $E = -8(2) = -16\ \text{Vm}^{-1}$

Here, negative sign shows that, it is in the direction along negative Z-axis.

Hence, option (b) is correct.

25. *(e)* Given, $q = 10\ \mu\text{C}$ and $x = 3$ cm

The work done in moving a point charge along the equatorial axis of an electric dipole is

$$W = -\mathbf{p}\cdot\mathbf{E} = -pE\cos\theta$$

$$= 0 \ (\because \text{at equatorial axis, } \theta = 90^\circ)$$

26. *(e)* When a steady current flows through a metallic conductor of non-uniform cross-section, then drift velocity,

$$v_d = \frac{I}{enA} \Rightarrow v_d \propto \frac{1}{A}$$

Electric field, $E = \frac{I}{\sigma A} \Rightarrow E \propto \frac{1}{A}$

Hence, both v_d and E change with cross-sectional area A. Only current I remains constant.

Hence, option (e) is correct.

27. *(e)* Let T be the temperature of hot water bath.

Given, $R_0 = 4\ \Omega$

$R_{100} = 4.25\ \Omega$

$R_T = 4.5\ \Omega$

As, $$\alpha = \frac{R_{100} - R_0}{R_0 \times 100} = \frac{(R_T - R_0)}{R_0 \times T}$$

$$\therefore \quad T = \frac{R_T - R_0}{R_{100} - R_0} \times 100$$

$$= \frac{4.5 - 4}{4.25 - 4} \times 100$$

$$= \frac{0.5}{0.25} \times 100 = \frac{50}{25} \times 100$$

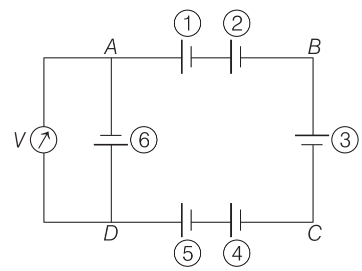
$T = 200^\circ\text{C}$

28. *(a)* The internal resistance of a cell depends on

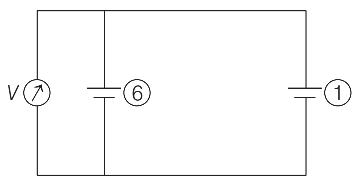
- Surface area of its electrodes
- Separation between its electrodes
- Nature, concentration and temperature of electrolyte.

Hence, internal resistance of a cell is independent of the circuit elements connected to it.

29. *(d)* In the loop *ABCD*, the cells (2) and (3) have same polarity but cells (4) and (5) have opposite polarity with respect to (2) and (3).



So, they cancel each other and the circuit is reduced to,



$\therefore$ Net emf of the circuit = 4(5V) − 2(5V)

Now, as cells (1) and (6) are also in reverse polarity, so the net emf or voltmeter reading is

$$V = 5 - 5 = 0\ \text{V}$$

Hence, option (d) is correct.

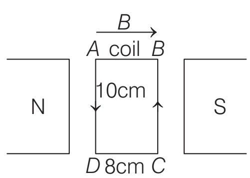
30. *(d)* According to Curie's law, the susceptibility of paramagnetic substances is inversely proportional to absolute temperature, i.e.

$$\chi_m \propto \frac{1}{T}$$

So, the susceptibility of paramagnetic materials decreases with rise in temperature.

31. *(a)* Given, $N = 50$, $I = 2\text{A}$, $B = 0.5\ \text{T}$

$$A = 10 \times 8 = 80\ \text{cm}^2 = 80 \times 10^{-4}\ \text{m}^2$$



As shown in figure, the sides *AB* and *DC* are along the magnetic field lines, so the torque acting on these sides will be zero.

$\therefore$ Net torque acting on the coil,

$$\tau = NIAB\sin\theta$$

Here, $\theta = 90^\circ$, for sides *AD* and *BC*.

$$\therefore \quad \tau = NIAB \quad [\because \sin 90^\circ = 1]$$

$$= 50 \times 2 \times 80 \times 10^{-4} \times 0.5$$

$$= 0.4\ \text{N-m}$$

According to Fleming's left hand rule, the force on side *BC* is downward and on *AD* is upward.

So, the coil will rotate in clockwise direction.

32. *(c)* Given, $n = \frac{N}{l} = 500$

$$I = 1.5\ \text{A}$$

Magnetic induction at one of the ends of the solenoid on its axis is given by

$$B = \frac{\mu_0 nI}{2}$$

$$= \frac{4\pi \times 10^{-7} \times 500 \times 1.5}{2}$$

$$= 47.1 \times 10^{-5}\ \text{T}$$

$$\approx 47 \times 10^{-5}\ \text{T}$$

33. *(b)* In most of the northern hemisphere, the North pole of the dip needle tilts downwards. Likewise, in most of the southern hemisphere, the South pole of the dip needle tilts downwards.

34. *(a)* Magnetic field at the centre of a circular coil is given by

$$B = \frac{\mu_0 NI}{2r}$$

Here, $N = 50$

$I = 1\text{A}$

and $r = 10\ \text{cm} = 0.1\text{m}$

$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 50 \times 1}{2 \times 0.1}$$

$$= 2\pi \times 10^{-7} \times 500 = 1000\ \pi \times 10^{-7}$$

$$\text{or} \quad B = \pi \times 10^{-4}\ \text{T}$$

35. *(e)* In a pure inductive circuit, the current lags behind the voltage by a phase angle of $\frac{\pi}{2}$ radians or the voltage leads the current by a phase angle of $\frac{\pi}{2}$ radians.

So, statement (e) is wrong.

36. (a) Given, $v = 72\text{ km h}^{-1} = 72 \times \frac{5}{18} = 20\text{ ms}^{-1}$

$$L = 150\text{ cm} = 1.5\text{ m}$$
$$B = 4.0 \times 10^{-5}\text{T}$$

$\therefore$ Induced emf on the rails,

$$E = BvL$$
$$= 4 \times 10^{-5} \times 20 \times 1.5$$
$$= 120 \times 10^{-5}\text{V}$$
$$= 1 \cdot 2 \times 10^{-3}\text{V}$$
$$= 1.2\text{ mV}$$

37. (b) Given, $V_p = 6\text{ kV} = 6 \times 10^3\text{V}$

$$V_s = 220\text{V}$$
$$P_s = 7.2\text{ kW} = 7.2 \times 10^3\text{W}$$
$$I_p = ?$$

As, $P_s = I_s V_s$

$\Rightarrow$ $I_s = \frac{P_s}{V_s} = \frac{7.2 \times 10^3}{220}$

$\Rightarrow$ $I_s = \frac{720}{22}\text{A}$

Now, for a transformer,

$$\frac{I_p}{I_s} = \frac{V_s}{V_p}$$

$\Rightarrow$ $I_P = \frac{V_S}{V_P} \times I_S$

$$= \frac{220}{6 \times 10^3} \times \frac{720}{22} = \frac{12}{10} = 1.2\text{ A}$$

38. (a) Emf induced in the circuit,

$$E = \frac{\Delta\phi_B}{\Delta t}$$

Induced current,

$$i = \frac{E}{r} = \frac{\Delta\phi_B}{R\Delta t}$$

Now, charge flowing in the circuit,

$$\Delta Q = i \times \Delta t = \frac{\Delta\phi_B}{r\Delta t} \times \Delta t$$

$\Rightarrow$ $\Delta Q = \frac{\Delta\phi_B}{r}$

39. (a) Frequency remains unchanged with change of medium.

Speed of wave in vacuum,

$$v_{\text{air}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$$

Speed of wave in medium,

$$v_{\text{medium}} = \frac{1}{\sqrt{\varepsilon_r\mu_r \times \mu_0\varepsilon_0}} = \frac{1}{\sqrt{\varepsilon_r}} \cdot c \quad (\because \mu_r = 1)$$
$$= \frac{c}{\sqrt{4}} = \frac{c}{2}$$

$\therefore$ $\frac{\lambda_{\text{medium}}}{\lambda_{\text{air}}} = \frac{v_{\text{medium}}}{v_{\text{air}}}$

$$= \frac{\frac{c}{2}}{c} = \frac{1}{2}$$

So, wavelength gets halved with change of medium.

40. (e) Exposure to UV-radiation induces the production of more melanin, causing tanning of the skin.

Due to its shorter wavelength, UV-radiations can be focussed into very narrow beams for high precision applications such as LASIK eye surgery.

UV lamps are used to kill germs in water purifiers.

X-rays are used in treatment of certain forms of cancer.

41. (e) Refractive index

$$(\mu) = \frac{\text{Speed of light in vacuum } (c)}{\text{Speed of light in medium } (v)}$$

$\Rightarrow$ $\mu \propto \frac{1}{v}$

Hence, when the refractive index increases, the wave speed decreases.

$\therefore$ Statement (e) is wrong.

42. (a) Angular width $= \frac{\text{Fringe width}}{D}$

$$= \frac{\frac{\lambda D}{a}}{D}$$

$\Rightarrow$ Angular width $= \frac{\lambda}{a}$

43. (b) Given, angle of refraction, $r = 30^\circ$

When reflected wave is perpendicular to the refracted wave, the reflected wave is completely polarised. In this case, the angle of incidence is called Brewster's angle (i_B).

Now, $i_B + r = \frac{\pi}{2}$

$\Rightarrow$ $i_B = \frac{\pi}{3}$ $\quad \left(\because r = 30^\circ = \frac{\pi}{6}\right)$

From Snell's law,

$$\mu = \frac{\sin i_B}{\sin r} = \frac{\sin\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{6}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$\mu = \sqrt{3} \Rightarrow \mu = 1.732$$

44. *(d)* For the same deviation i and e are exchangable.

So, for $\delta = 45^\circ$, $i = 43^\circ$ and $e = 62^\circ$

$$\therefore \quad A = i + e - \delta$$
$$= 43^\circ + 62^\circ - 45^\circ$$
$$\Rightarrow \quad A = 60^\circ$$

Now, $\delta_{min} = 42^\circ$

At minimum deviation, $i = e$

$$\Rightarrow \quad \delta_{min} = 2i - A$$
$$\Rightarrow \quad 42^\circ = 2i - 60^\circ$$
$$\Rightarrow \quad 2i = 102^\circ \Rightarrow i = 51^\circ$$

45. *(a)* Here, $I_1 = I$ and $I_2 = 4I$

Maximum intensity,

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$
$$= (\sqrt{I} + \sqrt{4I})^2$$
$$= (\sqrt{I} + 2\sqrt{I})^2 = (3\sqrt{I})^2$$
$$I_{max} = 9I$$

Minimum intensity,

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$
$$= (\sqrt{I} - \sqrt{4I})^2$$
$$= (\sqrt{I} - 2\sqrt{I})^2$$
$$I_{min} = I$$

Ratio, $\dfrac{I_{max}}{I_{min}} = \dfrac{9I}{I} = \dfrac{9}{1}$

46. *(c)* Amongst the given photosensitive substances, sodium is an alkali metal having only one valence electron in its outermost orbit. So, even by absorbing small amount of energy, i.e. from visible light, this valence electron leaves the atom of Na and is expelled out.

47. *(b)* The de-Broglie wavelength associated with matter wave is given by

$$\lambda = \frac{h}{\sqrt{2mK}}$$

where, h = Planck's constant,

m = mass of particle

and K = kinetic energy of particle.

As, in motion under gravity, the total energy of the particle is conserved. So, at the ground, the kinetic energy of the particle is equal to the potential energy of particle at height x.

$$\therefore \quad \lambda = \frac{h}{\sqrt{2m \times mgx}} \Rightarrow \lambda \propto \frac{1}{\sqrt{x}}$$

48. *(a)* The radioactive decay chain of ${}^{226}_{88}\text{Ra}$ to ${}^{206}_{82}\text{Pb}$ is shown below

$${}^{226}_{88}\text{Ra} \longrightarrow {}^{206}_{82}\text{Pb} + n\,{}^{4}_{2}\alpha + m\,{}^{0}_{-1}\beta$$

The alpha particles are helium particles while the β particles are electrons. So, equating mass and atomic number both the sides of chemical equation, we get

$$206 + 4n + 0 = 226$$
$$\Rightarrow \quad 4n + 0 = 20 \Rightarrow n = 5$$
$$\text{and} \quad 82 + 2n - m = 88$$
$$2n - m = 6$$
$$\Rightarrow \quad m = 10 - 6 = 4$$

$\therefore$ 5α-particles (${}^{4}_{2}\alpha$) and 4β-particles (${}^{0}_{-1}\beta$) are emitted in this process. So, correct option is (a).

49. *(d)* The wavelength of Paschen series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \text{ where } n = 4, 5, 6, \ldots\infty$$

For shortest wavelength, $n = \infty$

$$\frac{1}{\lambda_S} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = \frac{R}{9}$$

$$\Rightarrow \quad \lambda_S = \frac{9}{R} \quad \ldots \text{(i)}$$

For first member, $n = 4$

$$\frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R}{144}$$

$$\Rightarrow \quad \lambda_1 = \frac{144}{7R} \quad \ldots \text{(ii)}$$

Dividing Eqs. (ii) by (i), we get

$$\frac{\lambda_1}{\lambda_S} = \frac{144}{7R} \times \frac{R}{9} = \frac{16}{7}$$

$$\Rightarrow \quad \lambda_1 = \frac{16}{7}\lambda_S = \frac{16}{7} \times 8182$$

[Given $\lambda_S = 8182$Å]

$$\Rightarrow \quad \lambda_1 \simeq 18700\text{Å}$$

50. *(c)* In the given process, the momentum of the system remains constant or conserved i.e.,

Initial momentum = Final momentum

$$mu = m_1 v_1 + m_2 v_2$$

As initially the nucleus is at rest $(u = 0)$, so magnitude of momentum of two fragments are (neglecting – ve sign)

$$m_1 v_1 = m_2 v_2 \quad \dots (i)$$

The mass of a nucleus is given by,

$$m = \rho V = \rho \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad m \propto r^3$$

From Eq. (i).

$$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{r_2^3}{r_1^3}$$

Given, $\frac{r_1}{r_2} = \frac{2}{1}$

$$\therefore \quad \frac{v_1}{v_2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8} \text{ or } 1:8$$

51. *(c)* Given, amount of hydrogen, $m_H = 4$ kg

Amount of uranium, $m_U = 23.5$ kg

Energy released per fusion, $E_1 = 26$ MeV

Energy released per fission, $E_2 = 200$ MeV

1 mole or 1 g of hydrogen contains 6.023×10^{23} atoms.

Therefore, number of atoms in 4 kg is $6.023 \times 10^{23} \times 4 \times 10^3$.

In sun, 4 hydrogen atom fuses to give one ${}_2^4He$ atom.

$\therefore$ Energy released in fusion of 4 kg of hydrogen,

$$E_H = \frac{N \times m_H \times E_1}{A_H}$$

where, A_H = atomic mass of H-atom

$$E_H = \frac{6.023 \times 10^{23} \times 4 \times 10^3 \times 26}{4} \quad \dots (i)$$

Similarly, in fission of 23.5 kg of ^{235}U, the energy released is,

$$E_U = \frac{6.023 \times 10^{23} \times 200 \times 23.5 \times 10^3}{235} \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\frac{E_H}{E_U} = \frac{26 \times 4}{4} \times \frac{235}{200 \times 23.5}$$

$$= \frac{26 \times 10}{200} = \frac{13}{10} \text{ or } 13:10$$

52. *(a)* Initially, when both the diodes are connected in forward biased, then the current through the 2 kΩ resistor is

$$I = \frac{V_{eff}}{R}$$

Here, V_{eff} = effective voltage $= V - V_{Si} - V_{Ge}$

As we know, $V_{Si} = 0.7$V and $V_{Ge} = 0 \cdot 3$V and $V = 15$ V (given)

$$\Rightarrow \quad I_1 = \frac{15 - 0.7 - 0.3}{2 \times 10^3} = 7 \times 10^{-3} = 7\text{mA}$$

When, Ge diode is reverse biased it does not conduct, so the current becomes,

$$I_2 = \frac{15 - 0.7}{2 \times 10^3} = \frac{14.3}{2 \times 10^3} = 7.15 \text{ mA}$$

The change in current,

$$\Delta I = I_2 - I_1 = (7.15 - 7) = 0.15 \text{ mA}$$

$$\simeq 0.2 \text{ mA}$$

Hence, the current increases by nearly 0.2 mA.

53. *(e)* Given,

$$I_e = 0.75I$$

$$I_h = 0.25I$$

As we know,

$$I = neAv_d$$

where,

n = number of particles (electron or hole)

e = charge on particle

A = area of conductor

and v_d = drift velocity of particle

$$\Rightarrow \quad I \propto nv_d$$

$$\Rightarrow \quad \frac{I_e}{I_h} = \frac{n_e (v_d)_e}{n_h (v_d)_h}$$

Given, $(v_d)_e = \frac{3}{2}(v_d)_h$

$$\Rightarrow \quad \frac{I_e}{I_h} = \frac{3n_e (v_d)_h}{2n_h (v_d)_h} = \frac{3n_e}{2n_h}$$

$$\Rightarrow \quad \frac{n_e}{n_h} = \frac{2}{3} \times \frac{I_e}{I_h} = \frac{2}{3} \times \frac{0 \cdot 75}{0 \cdot 25} = \frac{2}{1} \text{ or } 2:1$$

54. *(b)* Given, $R_i = 200\ \Omega$, $R_0 = 500\ \Omega$,

Voltage gain = 50

As power, $P = VI = \frac{V^2}{R}$

$$\therefore \text{Power gain} = \frac{\text{Output power}}{\text{Input power}}$$

$$= (\text{Voltage gain})^2 \times \frac{\text{Input resistance}}{\text{Output resistance}}$$

$$= (50)^2 \times \frac{200}{500} = 2500 \times \frac{2}{5}$$

$$= 1000$$

55. *(d)* The output of gates for two inputs ($A = 1$ and $B = 1$) are given below:

AND : $Y = A \cdot B = 1$

OR : $Y = A + B = 1$

NAND : $Y = \overline{A \cdot B} = 0$

NOR : $Y = \overline{A + B} = 0$

Hence, correct option is (d).

56. *(c)* Given, audio frequency, $f_m = 700$ Hz.

The range of frequency for AM is from (540 – 1600) kHz.

Also, carries frequency > audio frequency

So, the correct option is (c) i.e., $f_c = 1000$ kHz.

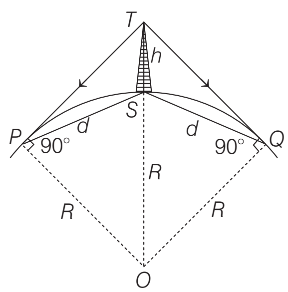
57. *(e)* Suppose, ST is a space wave transmitting antenna of height h. Let $SP = QS = d$ be the distance of horizon. This distance is limited by the curvature of the earth.

Also, R = radius of the earth

$= OP = OQ = OS$

In right angled ΔTSP,

$$PT^2 = PS^2 + ST^2 = d^2 + h^2$$



Similarly, in right angled ΔOPT,

$$OT^2 = OP^2 + PT^2$$

$\Rightarrow \quad (R + h)^2 = R^2 + (d^2 + h^2)$

$\Rightarrow R^2 + 2hR + h^2 = R^2 + d^2 + h^2$

$[\because (a + b)^2 = a^2 + 2ab + b^2]$

$\Rightarrow \quad 2hR = d^2$

$\Rightarrow \quad d = \sqrt{2hR}$

The maximum line of sight distance d_M between two transmitting antenna of height h_T and receiving antenna of height h_R above the earth is given by

$$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

58. *(d)* The dimensions of electrical permittivity,

$$\varepsilon_0 = [M^{-1}L^{-3}T^4A^2]$$

and that of magnetic permeability is,

$$\mu_0 = [MLT^{-2}A^{-2}]$$

$\therefore$ Dimensional formula for $\dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$= \frac{1}{[MLT^{-2}A^{-2}]^{1/2}[M^{-1}L^{-3}T^4A^2]^{1/2}}$$

$$= [M^0LT^{-1}]$$

59. *(d)* Given,

$$I = 5 \pm 0.4A$$

$$V = 10 \pm 0.2V$$

and $\quad \dfrac{\Delta P}{P} \times 100 = 10\%$

As we know,

Power, $P = VI$

$\Rightarrow \quad \dfrac{\Delta P}{P} \times 100 = \dfrac{\Delta V}{V} \times 100 + \dfrac{\Delta I}{I} \times 100$

For, $\dfrac{\Delta P}{P} \times 100 = 5\%$, the error is measurement of current should be,

$$\frac{\Delta I}{I} \times 100 = \frac{\Delta P}{P} \times 100 - \frac{\Delta V}{V} \times 100$$

$$= 5 - \frac{0.2}{10} \times 100 = 5 - 2 = 3\%$$

60. *(b)* Given, angular diameter of planet from earth,

$\theta = 90'' = \dfrac{90}{3600} \times \dfrac{\pi}{180}$ rad

Diameter of planet, $D = \pi \times 10^6$m

$\therefore$ Distance of planet from earth is,

$$d = \frac{D}{\theta} = \frac{\pi \times 10^6}{90 \times \pi} \times 3600 \times 180$$

$$= 7200 \times 10^6 \text{ or } 7.2 \times 10^9 \text{ m}$$

61. *(e)* The resultant of $(2\mathbf{A} + 3\mathbf{B})$ and $(4\mathbf{A} - 3\mathbf{B})$ is,

$$\mathbf{R} = (2\mathbf{A} + 3\mathbf{B}) + (4\mathbf{A} - 3\mathbf{B})$$

$$= 6\mathbf{A}$$

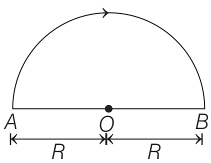
$\therefore$ Angle between $\mathbf{A}$ and $\mathbf{R}$,

$$\cos\theta = \frac{\mathbf{A} \cdot \mathbf{R}}{|\mathbf{A}||\mathbf{R}|}$$

$$= \frac{6(\mathbf{A} \cdot \mathbf{A})}{6|\mathbf{A}||\mathbf{A}|} = \frac{A^2}{A^2} = 1$$

$\Rightarrow \quad \theta = \cos^{-1}(1) = 0^\circ$

62. *(c)* The given situation is shown below :



Here, R = radius of semicircle

Distance travelled between A and $B = \pi R$

Magnitude of displacement between A and $B = 2R$

As, distance $\neq$ displacement $\neq 0$

So, average velocity and average acceleration cannot be zero and average velocity and average speed are not equal.

63. *(b)* The horizontal range in projectile motion is given by,

$$R = \frac{u^2 \sin 2\theta}{g}$$

Give, $R_P = R_Q$

$$\Rightarrow \quad \frac{u_P^2 \sin 2\theta_P}{g} = \frac{u_Q \sin 2\theta_Q}{g} \qquad \ldots \text{(i)}$$

Given, $u_P = v$, $u_a = \frac{v}{2}$ and $\theta_Q = 15^\circ$

Substituting these values in Eq. (i), we get

$$v^2 \sin 2\theta_P = \frac{v^2}{4} \sin(2 \times 15^\circ)$$

$$\Rightarrow \quad \sin 2\theta_P = \frac{1}{4} \sin 30^\circ$$

$$= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$\Rightarrow \quad \theta_P = \frac{1}{2} \sin^{-1}\left(\frac{1}{8}\right)$$

64. *(c)* When the object is moving upward with velocity u, then acceleration of object is,

$$g_1 = \frac{v_1 - u_1}{t_1}$$

At highest point, $v_1 = 0$ and $u_1 = u$ (given)

$$\Rightarrow \quad g_1 = \frac{0 - u}{t_1} = \frac{-u}{t_1} \qquad \ldots \text{(i)}$$

When the object is moving down, then

$$g_2 = \frac{v_2 - u_2}{t_2}$$

Here, $u_2 v_1 = 0$,

$$\Rightarrow \quad g_2 = \frac{v_2 - 0}{t_2} = \frac{v_2}{t_2} \qquad \ldots \text{(ii)}$$

As, acceleration due to gravity (g) is same on object and distance is also same. So, time will also be same (say t) for equal distance.

From Eqs (i) and (ii), we get

$$-\frac{u}{t} = \frac{v_2}{t} \quad \Rightarrow \quad v_2 = -u$$

Hence, the object will strike the ground with velocity $-u$.

65. *(a)* Only statement given in option (a) is correct, rest are incorrect and can be corrected as,

(b) Second law of motion is applicable to a particle and also to a system of particles. As, the applied force causes a change in momentum of a particle and also to a system of particles.

(c) In case of a circular motion, the motion of object is along tangent, while the centripetal force is along the centre of circle. So, force is not always in the direction of motion.

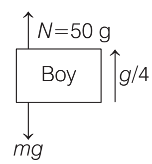
(d) As, force, $F = ma$

Mass of a body cannot be zero.

So, if $F = 0 \Rightarrow a = 0$

(e) The acceleration of a particle at an instant is called instantaneous acceleration, and depends on the value of acceleration of the particle at that instant of time and not on its history.

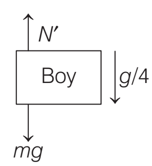
66. *(b)* When the lift is going upwards with acceleration $\frac{g}{4}$, machine reading is 50 kg-wt.



$$\Rightarrow \quad N - mg = m\left(\frac{g}{4}\right) \quad \Rightarrow \quad N = \frac{5mg}{4}$$

$$\Rightarrow \quad 50\,g = \frac{5mg}{4} \quad \Rightarrow \quad m = 40 \text{ kg}$$

While going down with acceleration $g/4$, the reaction would be



$$mg - N' = m\left(\frac{g}{4}\right)$$

$\Rightarrow \quad \frac{3mg}{4} = N' \Rightarrow N' = \frac{3 \times 40}{4} g$

$\Rightarrow \quad N' = 30g$

$\therefore$ Reading of machine = 30 kg-wt

67. (*) Given, $m = 2 \times 10^7$ kg

$u = 0$

$F = 5 \times 10^5$ N

$S = 2$m

$v = ?$

Acceleration, $a = \frac{F}{m} = \frac{5 \times 10^5}{2 \times 10^7} = \frac{5}{2} \times 10^{-2} \text{ms}^{-2}$

From 3rd equation of motion,

$$v^2 - u^2 = 2as$$

$$\Rightarrow \quad v^2 - 0 = 2 \times \frac{5}{2} \times 10^{-2} \times 2$$

$$\Rightarrow \quad v^2 = 10 \times 10^{-2} = 10^{-1}$$

$$\Rightarrow \quad v = \sqrt{0.1} = 0.316 \text{ ms}^{-1}$$

Hence, no option matches the answer.

68. (b) Given, $\mathbf{F} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})N$

$m = 1$ kg

$\mathbf{a} = ?$

From Newton's second law of motion,

$$\mathbf{F} = m\mathbf{a}$$

$$\Rightarrow \quad \mathbf{a} = \frac{\mathbf{F}}{m} = \frac{(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})}{1}$$

$$\mathbf{a} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \text{ ms}^{-2}$$

69. (c) Work done is given by

$$W = \int \mathbf{F} \cdot d\mathbf{s}$$

Using Newton's second law, $\mathbf{F} = m\mathbf{a}$, we get

$$W = \int m\mathbf{a} \cdot d\mathbf{s}$$

$$= \int m\frac{d\mathbf{v}}{dt} \cdot d\mathbf{s}$$

$$= \int m d\mathbf{v} \cdot \frac{d\mathbf{s}}{dt}$$

$$= \int_{v_i}^{v_f} m\mathbf{v}\, dv$$

$$W = \frac{1}{2}m(v_f^2 - v_i^2)$$

From work - energy theorem, the total work done is equal to the change in kinetic energy. As this theorem deals with scalar quantities and can be derived from Newton's second law, hence it may be viewed as scalar form of Newton's second law.

70. (c) Let, m be the mass of the boy, then mass of man will be $2m$.

Given, speed of boy, $v_B = 14.14 \text{ ms}^{-1}$

Let v_m be the speed of man.

Given that,

KE of man = KE of boy

$$\Rightarrow \quad \frac{1}{2}(2m)v_m^2 = \frac{1}{2} mv_B^2$$

$$\Rightarrow \quad 2v_m^2 = v_B^2$$

$$\Rightarrow \quad v_m^2 = \frac{v_B^2}{2}$$

$$\Rightarrow \quad v_m = \frac{v_B}{\sqrt{2}} = \frac{14.14 \text{ ms}^{-1}}{\sqrt{2}}$$

$$= \frac{14.14}{1.414} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_m = 10 \text{ ms}^{-1}$$

71. (c) Given, $m = 2$ kg

$p = 10$ kg - ms^{-1}

$t = 10$ s

Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{1}{2m}m^2v^2$

$$K = \frac{p^2}{2m}$$

$$\Rightarrow \quad K \propto p^2$$

So, to increase the kinetic energy by four times, momentum should get doubled.

$$\therefore \quad p' = 2p = 20 \text{ kg - ms}^{-1}$$

From Newton's second law of motion,

$$F = \frac{\text{Change in momentum}}{\text{Time}}$$

$$= \frac{p' - p}{t} = \frac{20 - 10}{10}$$

$$\Rightarrow \quad F = 1\text{N}$$

72. (e) Given, force $\mathbf{F} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

Displacement, $\mathbf{r} = (2\hat{\mathbf{i}} - \hat{\mathbf{j}}) - (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$$\mathbf{r} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

Work done by the force,

$$W = \mathbf{F} \cdot \mathbf{r} = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}) \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$= 1 + 4 + 4$$

$$W = 9 \text{ (units)}$$

Chemistry

73. *(d)* (d) Entropy decreases when water crystallises into ice. After freezing, the molecules attain an ordered state and therefore, entropy decreases.

Entropy increases for reaction (a), (b), (c) and (e). Explanation is as follows:

(a) When sodium chloride dissolves in water, it gets ionised into Na^+ and $Cl^{\ominus}$. As a result, number of ion increases. Hence, entropy increases.

(b) Water is heated from 303 K to 353 K. As temperature is raised to 353 K, molecules begin to move and oscillate about their equilibrium positions in the lattice.

(c) $NaHCO_3$ is a solid and it has low entropy. Among products there are one solid and two gases. Therefore, products represent a condition of higher entropy.

(e) Increase in the number of gas molecules, increases entropy.

74. *(a)* The formation reaction of methane is given by

$$C(s) + 2H_2(g) \longrightarrow CH_4(g);\ \Delta_f H^{\ominus} = ?$$

The standard heat of combustion of $CH_4(g)$ is

$$CH_4(g) + 2O_2(g) \longrightarrow CO_2(g) + 2H_2O(l);\ \Delta H^{\ominus} = -891\,\text{kJ mol}^{-1} \quad \ldots(A)$$

The enthalpy of formation of 1 mol of $CO_2(g)$

$$C(\text{Graphite}) + O_2(g) \longrightarrow CO_2(g); \quad \ldots(i)$$

$$\Delta_f H^{\ominus} = -394\,\text{kJ mol}^{-1}$$

The enthalpy of formation of 1 mole of $H_2O(l)$ is

$$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l);$$

$$\Delta_f H^{\ominus} = -286\,\text{kJ mol}^{-1} \quad \ldots(ii)$$

Multiplying Eq. (ii) by 2

$$2H_2(g) + O_2(g) \longrightarrow 2H_2O(l); \quad \ldots(iii)$$

$$\Delta_f H^{\ominus} = -572\,\text{kJ mol}^{-1}$$

Adding Eqs. (i) and (iii)

$$C(\text{Graphite}) + 2O_2(g) + 2H_2(g) \longrightarrow CO_2(g) + 2H_2O(l);$$

$$\Delta_f H^{\ominus} = -966\,\text{kJ mol}^{-1} \quad \ldots(iv)$$

Reversing equation (A)

$$CO_2(g) + 2H_2O(l) \longrightarrow CH_4(g) + 2O_2(g);$$

$$\Delta_f H^{\ominus} = +891\,\text{kJ mol}^{-1} \quad \ldots(v)$$

Adding Eqs (iv) and (v), we get

$$C(\text{Graphite}) + 2H_2(g) \longrightarrow CH_4(g);$$

$$\Delta_f H^{\ominus} = -75\,\text{kJ mol}^{-1}$$

75. *(b)* Complete reaction

	$PCl_5(g) \rightleftharpoons$	$PCl_3(g)$	$+ Cl_2(g)$
No. of moles initial	x	0	0
No. of moles at equilib.	$x - 0.2$	0.2	0.2

So, equilibrium constant, $k = \dfrac{[PCl_3][Cl_2]}{[PCl_5]}$

Given, $k = 2 \times 10^{-2}$ mol/L

$$\therefore\ 2 \times 10^{-2} = \frac{0.2 \times 0.2}{x - 0.2} \Rightarrow x = 2.2\ \text{mol}$$

76. *(d)* Number of milliequivalents of HCl $= 10^{-2} \times 20$

Normality of $Ca(OH)_2 = M \times$ Acidity

$= 0.005 \times 2 = 0.01$

Number of milliequivalents of $Ca(OH)_2 = 10^{-2} \times 20$

In a reaction, where number of equivalents of acid (HCl) is equal to the number of equivalents of base ($Ca(OH)_2$) then solution is neutral and the pH of the solution is 7.

77. *(c)* In the given reaction, the change of oxidation state of carbon is form − 4 to + 4.

$$\overset{-4}{C}H_4(g) + 4Cl_2(g) \longrightarrow \overset{+4}{C}Cl_4(l) + 4HCl(g)$$

For CH_4, $x + 4(+1) = 0 \Rightarrow x = -4$

For CCl_4, $x + 4(-1) = 0 \Rightarrow x = +4$

78. *(e)* $Pt^{4+} + 4e^- \longrightarrow Pt$

4F of electricity ≡ 1 mol Pt

When 0.40 F of electricity is passed through solution then number of moles of Pt = x

So, number of moles $(x) = \dfrac{0.4\,F}{4\,F} = 0.10$ mol

79. *(c)* Given, $[M_B]_P = 80\ \text{g mol}^{-1}$

$(\Delta T_f)_P = 0.15\,K$

$(\Delta T_f)_Q = 0.30\,K$

As we know that,

$$\Delta T_f \propto \frac{1}{M_B\ (\text{Molecular mass of solute})}$$

$$\therefore \frac{(\Delta T_f)_P}{(\Delta T_f)_Q} = \frac{[M_B]_Q}{[M_B]_P}$$

$$\frac{0.15\,K}{0.30\,K} = \frac{[M_B]_Q}{80\ \text{g mol}^{-1}}$$

$$0.5 \times 80 \text{ g mol}^{-1} = [M_B]_Q$$

$$40 \text{ g mol}^{-1} = [M_B]_Q$$

So, the molecular weight of 'Q' is 40 g mol^{-1}.

80. *(e)* Given, $w_B = 20$ g

$$M_B = 40 \text{ g}$$

$$w_A = d \times V = 0.8 \text{ g mL}^{-1} \times 1250 \text{ mL}$$

$$= 1000 \text{ g} = 1 \text{ kg}$$

$$M = \frac{n_B}{w_A(\text{kg})} = \frac{w_B}{M_B \times w_A(\text{kg})}$$

$$= \frac{20}{40 \times 1} = 0.5 \text{ mol kg}^{-1}$$

81. *(b)* Given, $4\text{g} \xrightarrow{t_{1/2}} 2\text{g}$

For first order reaction

$$t_{1/2} = \frac{0.693}{k} = \frac{0.693}{231 \times 10^{-5}}$$

$$t_{1/2} = 300 \text{ s}$$

82. *(b)* $A \rightarrow B$ is a forward reaction whereas $B \rightarrow A$ is a backward reaction.

Given, $Ea = 13 \text{ kJ mol}^{-1}$, $\Delta H = 2 \text{ kJ mol}^{-1}$

$$\Delta H = Ea_{(f)} - Ea_{(b)}$$

$$2 \text{ kJ mol}^{-1} = 13 \text{ kJ mol}^{-1} - Ea_{(b)}$$

$$2 \text{ kJ mol}^{-1} - 13 \text{ kJ mol}^{-1} = Ea_{(b)}$$

$$-11 \text{ kJ mol}^{-1} = Ea_{(b)}$$

Hence, activation energy of the reaction $B \rightarrow A$ is 11 kJ mol^{-1}.

83. *(a)* Adsorption is accompanied by decrease in enthalpy and decrease in entropy.

During adsorption, there is always decrease in surface energies. This decrease in surface energy appears as heat, therefore adsorption is an exothermic process, i.e. ΔH of adsorption is always negative. As the molecules of the gas are held on the surface of the solid adsorbent hence entropy decreases i.e. ΔS is also negative.

84. *(b)* In the coagulation of a positive sol, the flocculating power of the given ions decreases in the order

$$PO_4^{3-} > SO_4^{2-} > Cl^-$$

According to hardly Schulze rule, greater the valence of the flocculating ion added, the greater is its power to cause precipitation. The degree of coagulation is directly proportional to the valency of the effective ion used.

85. *(e)* Potassium nitrate does not give metallic oxide, nitrogen dioxide and oxygen on heating. It decomposes to nitrite and oxygen whereas all alkaline earth metals and lithium decomposes to metal oxide, dinitrogen and oxygen.

$$KNO_3 \xrightarrow{\Delta} KNO_2 + O_2 \uparrow$$

$$MNO_3 \xrightarrow{\Delta} M_2O + NO_2 + O_2 \uparrow$$

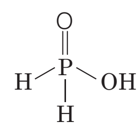
(M = Alkaline earth metal + Lithium)

86. *(d)* Statement (d) is incorrect whereas all other statements are correct. Corrected statement is as follows:

Beryllium shows a maximum coordination number of four in its compounds. This is availability of vacant d-orbitals in the valence shell.

$$\text{Be}(4) = 1s^2 2s^2 \text{ (Absence of } d\text{-orbital)}$$

87. *(a)* The oxyacid of phosphorus that contains one P—OH, two P—H and one P═O bonds is phosphinic acid. It's molecular formula is H_3PO_2. It is also known as hypophosphorus acid. It's structural formula is as follows:



(H_3PO_2) Hypophosphorus acid (Phosphinic acid)

88. *(b)* Statements I, II and V are correct whereas statements III, IV are incorrect. Diborane is prepared by the oxidation of sodium borohydride with iodine. It undergoes cleavage reactions with Lewis bases to give borane adducts. It is produced on an industrial scale by the reaction of BF_3 with sodium hydride (NaH).

$$2BF_3 + 6NaH \xrightarrow{450 \text{ K}} B_2H_6 + 6NaF$$

It is readily hydrolysed by water to give orthoboric acid (H_3BO_3) and hydrogen (H_2).
Diborane burns in oxygen and gives boron trioxide.

89. *(e)* Pu (Plutonium) has no electron in 6d orbital. The atomic number is 94 and configuration is [Rn] $5f^6$, $6d^0$, $7s^2$. Other given options such as Pa, Np, Lr and Cm has electrons in 6d orbital.

Element symbol	Name and atomic number	Electronic configuration
Pa	Protactinium (91)	[Rn]$5f^2$, $6d^1$, $7s^2$
Np	Neptunium (93)	[Rn]$5f^4$, $6d^1$, $7s^2$
Lr	Lawrencium (103)	[Rn]$5f^{14}$, $6d^1$, $7s^2$
Cm	Curium (96)	[Rn]$5f^7$, $6d^1$, $7s^2$

90. *(c)* The catalyst used in the Wacker process of oxidation of ethene to ethanal is $PdCl_2$. Main function of $PdCl_2$ is oxidation. Chemical reaction involved is as follows.

$$CH_2{=\!=}CH_2 + PCl_2 + H_2O \xrightarrow{CuCl_2} CH_3CHO + Pd + 2HCl$$

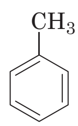
91. *(e)* The correct formula of dichlorobis (triphenylphosphine) nickel (II) is $[NiCl_2(PPh_3)_2]$. While writing the formula of a complex central atom is listed first followed by ligands in alphabetical order.

92. *(d)* $SCN^{\ominus}$ is an ambidentate ligand because it can ligate through two different atoms. It can coordinate through sulphur or nitrogen atom.

$M \leftarrow SCN$ Thiocyanato $\quad M \leftarrow NCS$ Isothiocyanato

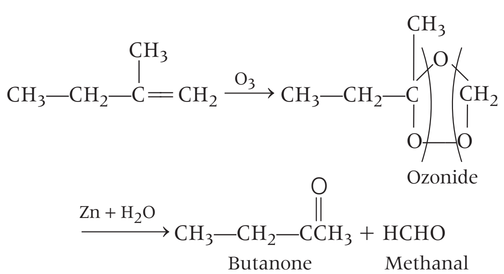
93. *(e)* Cuprite-$CuCO_3.Cu(OH)_2$ is not correctly matched. The correct match is cuprite – Cu_2O, whereas all other options are correctly matched.

94. *(d)* Toluene is a benzenoid aromatic compound because it has a benzene ring. It's structural formula is



Cyclooctatetraene is a non-benzenoid aromatic compound. Hexyne is an acyclic compound. Cyclohexane and cyclopentadiene an alicyclic compounds.

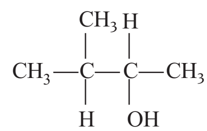
95. *(c)* The products obtained by the ozonolysis of 2-methylbut-1-ene are butanone and methanal. In this reaction, 2-methylbut-1-ene react with O_3 to give ozomide, which on reduction with $Zn—H_2O$ give smaller molecules.



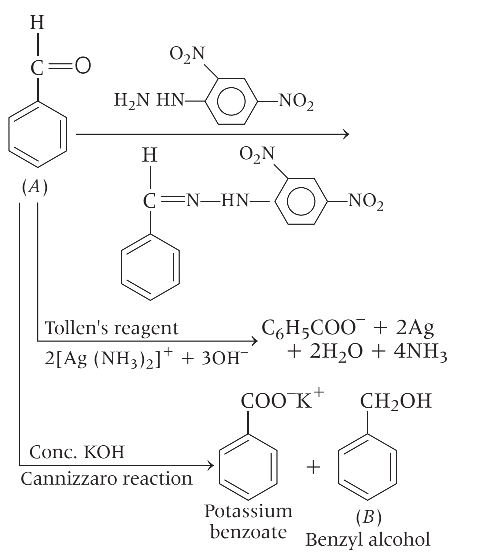
96. *(a)* 2, 3-dimethylbuta-1, 3-diene is not an isomer of 3-methylbut-1-yne because both compounds have different molecular formula. The molecular formula of 2, 3-dimethylbuta-1, 3-diene is C_6H_{10} whereas for 3-methylbut-1-yne is C_5H_9. Other given options such as pent-1-yne, pent-2-yne, penta-1, 3-diene, 2-methylbuta-1, 3-diene are isomers of 3-methylbut-1-yne as they have same molecular but different structural formula.

97. *(c)* Rate of S_N1 mechanism depends on stability of carbocation, greater the stability of carbocation, greater will be its ease of hydrolysis. The carbocation formed in case of C_6H_5Cl is $C_6H_5^{\oplus}$ which is highly unstable. So, compound C_6H_5Cl does not undergo hydrolysis by S_N1 mechanism.

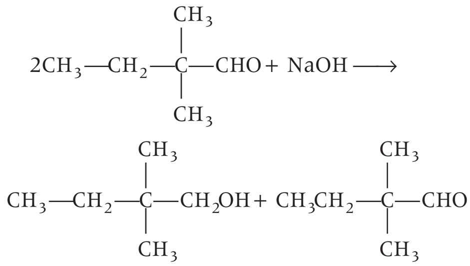
98. *(d)* 3-methylbutan-2-ol is a secondary alcohol. In this alcohol, the —OH group is attached to secondary carbon.



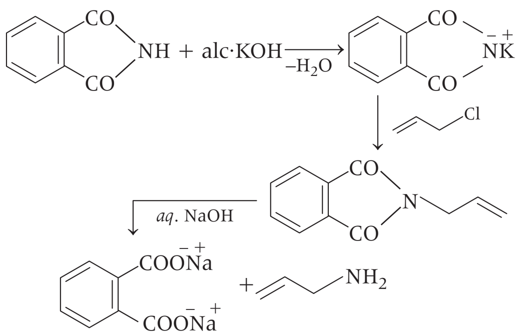
99. *(e)* Organic compound '*A*' is benzaldehyde that forms 2, 4-DNP derivatives and reduces Tollen's reagent. On reaction, it produces a bright silver mirror. The compound get oxidised to carboxylate anion. When '*A*' reacts with conc.KOH then potassium benzoate and benzyl alcohol '*B*' are formed. Chemical reactions involved are as follow:



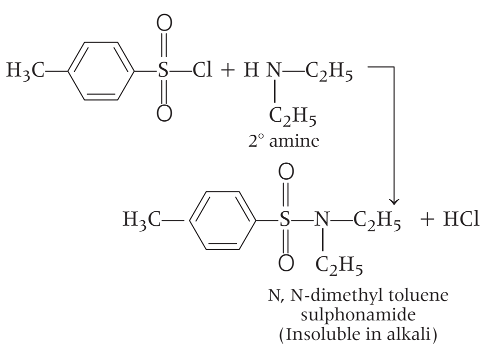
100. *(c)* 2, 2-dimethylbutanal would undergo Cannizzaro reaction because aldehyde does not have α-hydrogen atom. It undergoes disproportionation reaction in which half of the molecules of aldehyde are oxidised and other half are reduced.



101. *(d)* Allyl amine can be prepared by Gabriel phthalimide synthesis. Only primary aliphatic amines can be prepared by this method. In this method, phthalimide is treated with ethanolic KOH and forms potassium salt of phthalimide which on heating with alkyl halide followed by alkaline hydrolysis forms corresponding primary amines.



102. *(a)* The reagent used to distinguish between a secondary amine and tertiary amine is *p*-toluene sulphonyl chloride. Secondary amine when reacts with *p*-toluene sulphonyl chloride, it yields N, N-dimethyl toluene sulphonamide. It does not have any H-atom attached to N-atom, it is not acidic. Hence, insoluble in alkali.



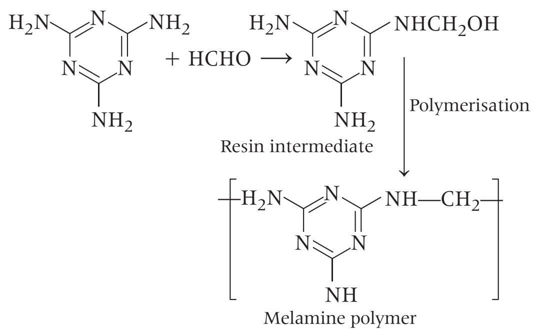
Tertiary amines do not react with benzene sulphonyl chloride.

103. *(e)* Statement (e) is correct whereas all other statements are incorrect. Corrected statements are as follows.

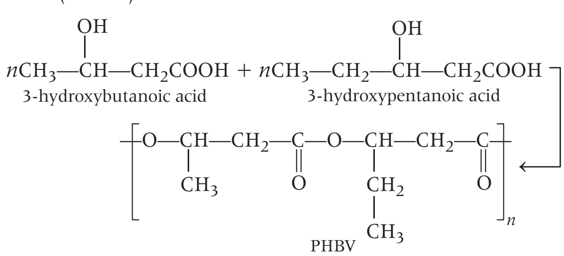
(a) Cellulose is also known as plant starch.
(b) A linkage between two monosaccharide units through oxygen atom is called glycosidic linkage.
(c) Glucose on oxidation with bromine gives gluconic acid.
(d) Glycogen are used as storage molecules as starch in animals.

104. *(c)* Sucrose is a non-reducing sugar. Here, aldehydic or ketonic group are bonded and do not reduce Fehling's solution or Tollen's reagent. Other options lactose, glucose, maltose and fructose are reducing sugars. In these sugars, ketonic or aldehydic groups are free and are capable of reducing Fehling's solution or Tollen's reagent.

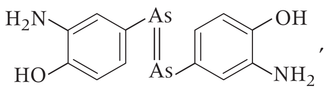
105. *(d)* Melamine-formaldehyde polymer is a copolymer formed by condensation polymerisation. In this particular reaction, melamine and formaldehyde copolymerise to give another polymer.



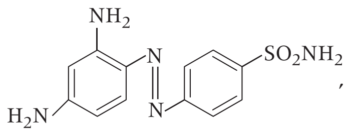
106. *(a)* 3-hydroxybutanoic acid and 3-hydroxy pentanoic acid forms the biodegradable polymer. Both these monomers result in the formation of poly β-hydroxy butyrate co-β-hydroxy valerate (PHBV). Reaction is as follows.



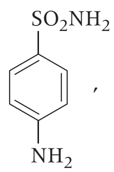
107. *(b)* The antimicrobial drug that contains arsenic is salvarsan. It contain —As ═ As —linkage. It is used for the treatment of syphilis. Structures of salvarsan and other given options is as follows.



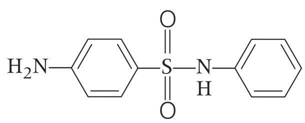
Salvarsan



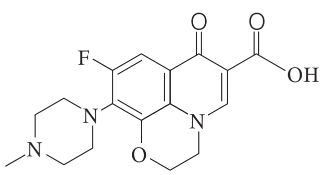
Prontosil



Sulphanilamide



Sulphapyridine



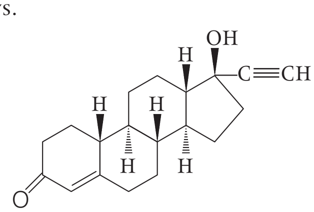
Ofloxacin

108. *(e)* Statement (e) is not correct whereas all other statements are correct. Correct statement is as follows:

Glucose on oxidation with bromine water gives gluconic acid. Reaction is as follows:

$$\underset{\text{D-glucose}}{CHO-(CHOH)_4-CH_2OH} \xrightarrow[\text{(Mild oxidising agent)}]{Br_2\ \text{water}} \underset{\text{Gluconic acid}}{COOH-(CHOH)_4-CH_2OH}$$

109. *(c)* Norethindrone is an antifertility drug. It is an example of synthetic progesterone derivative. These drugs are used to prevent unwanted pregnancies in woman. Structural formula is as follows.



Bithional and terpineol antiseptics.
Ofloxacin is an antibiotic.
Aspartame is artificial sweetener.

110. *(a)* Methane (CH_4) is a greenhouse gas. It is produced when vegetation is burnt, digested or rotted in the absence of oxygen. It is also released in paddy fields. Coalmines from rotting garbage dump and by fossil fuels. The gas traps the IR radiations and does not allow them to go out of the earth's atmosphere.

111. *(c)* (a) $1\text{ g Au} = \frac{1}{197}$ mole of Au

$= \frac{1}{197} \times 6.022 \times 10^{23}$ atoms of Au

$= 3.057 \times 10^{21}$ atoms

(b) $1\text{ g Na} = \frac{1}{23}$ mole of Na

$= \frac{1}{23} \times 6.022 \times 10^{23}$ atoms of Na

$= 2.6 \times 10^{22}$ atoms

(c) $1\text{g Li} = \frac{1}{7}$ mole of Li $= \frac{1}{7} \times 6.022 \times 10^{23}$ molecules of Li

$= 8.60 \times 10^{22}$ atoms

(d) $1\text{ g } Cl_2 = \frac{1}{71}$ mole of $Cl_2 = \frac{1}{71} \times 6.022 \times 10^{23}$ atoms Cl_2

$= 8.48 \times 10^{21}$ atoms

(e) $1\text{ g of } O_2(g) = \frac{1}{32}$ mole of $O_2 = \frac{1}{32} \times 6.022 \times 10^{23}$ atoms of O_2

$= 1.88 \times 10^{22}$ atoms

So, 1 g Li have the largest number of atoms.

112. *(b)* Calculation of the empirical formula:

Element	Percentage	Atomic mass	Gram atoms	Molar ratio	Simplest whole number ratio
C	24%	12	2	$\frac{2}{2} = 1$	1
H	4%	1	4	$\frac{4}{2} = 2$	2
Cl	72%	35.5	2	$\frac{2}{2} = 1$	1

∴The simplest whole number ratios of the different elements are C : H : Cl : : 1 : 2 : 1 and the empirical formula of the compound is CH_2Cl.

113. *(c)* The IUPAC name of an element is unbinilium. Its atomic number is 120. According to IUPAC nomenclature of inorganic chemistry, 'un' is used for '1', 'bi' is used for '2' and 'nil' is used for '0'. The suffix 'ium' is added at the end.

114. *(d)* Number of proton in the nucleus of an atom is equal to the atomic number. So, atomic number of species is 11. In a neutral atom, number of electrons are equal to the number of protons. But in this question number of electrons is one less than number of protons. So, it is an ionic species with +1 charge. The sum of neutrons and protons is known as mass number. So, for the given species.

Atomic number = 11

Mass number = 11 + 12 = 23

Hence, the proper symbol of the species is ${}^{23}_{11}Na^+$.

115. *(b)* Germanium is represented as Eka-silicon in Mendeleev's periodic table. He named the element because of resemblance in properties with silicon. He also predicted the properties of elements. A comparison in the properties of these elements as predicted by Mendeleev and those found experimentally are tabulated below:

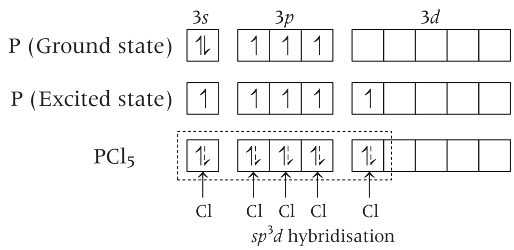
Property	Eka-silicon (predicted)	Germanium (found)
Atomic weight	72	72.6
Density	5.5	5.36
Melting point	High	1231
Formula of oxide	EO_2	GeO_2
Formula of chloride	ECl_4	$GeCl_4$

116. *(c)* Correct match is (A) → (III), (B) → (I), (C) → (IV), (D) → (II)

- Lithium (Li), sodium (Na), potassium (K) are alkali metals (Group 1). They dissolve in water to form hydroxides that are strongly alkaline in nature.
- Beryllium, magnesium and calcium are alkaline earth metals. (Group 2). Their oxides are alkaline in nature and are found in earth's crust.
- Oxygen, sulphur and selenium are chalcogens (Group 17). They readily forms ores.
- Silicon, germanium and arsenic are semi-metals (Metalloids). They are the mixture of both metals and non-metals.

117. *(b)* PF_5 molecule is formed by sp^3d hybridisation.

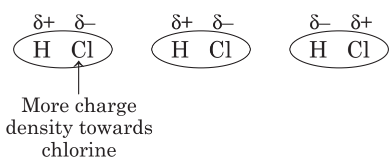
The ground state and the excited state outer electronic configurations of the central atom i.e. phosphorus ($Z = 15$) are given below:



118. *(d)* The correct order of bond energy is
$B_2 < O_2 < C_2 < N_2$

Nitrogen molecule contains triple bond, thus have highest bond energy followed by carbon which can have double or triple bond, then oxygen which can have only double bond.

119. *(a)* The type of attractive forces that operate between gaseous HCl molecules is dipole-dipole forces. These forces exist between the molecules having permanent dipoles. Ends of such dipoles have partial charges which are always less than the unit electronic charge. These forces arise because of the interactions between oppositely charged ends. The dipole-dipole interaction between HCl molecules is shown below.



120. *(b)* Schottky defect is shown by ionic substances in which the cation and anion are of almost similar sizes. They maintain electrical neutrality. This defect arises when equal number of cations and anions are missing from the lattice.

Mathematics

1. *(c)* We have,
$$f(x) = \sqrt{x-1}$$
$f(x)$ is defined if $x - 1 \geq 0 \Rightarrow x \geq 1$
$\therefore$ Domain of $f(x)$ is $[1, \infty)$.

2. *(d)* We have, $f(x) = -2x^2 + 1$ and $g(x) = 4x - 3$
$(gof)(x) = g(f(x)) = 4f(x) - 3 = 4(-2x^2 + 1) - 3$
$(gof)(x) = -8x^2 + 4 - 3 = -8x^2 + 1$
$(gof)(-1) = -8(-1)^2 + 1 = -8 + 1 = -7$

3. *(a)* Given, $n(A - B) = 18$, $n(A \cap B) = 25$, $n(A \cup B) = 70$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cup B) = n(A - B) + n(B)$
$[\because n(A - B) = n(A) - n(A \cap B)]$
$70 = 18 + n(B)$
$n(B) = 70 - 18 = 52$

4. *(c)* Given, $n(M \cup E) = 100$
$n(M) = 80$
$n(E) = 60$
$\therefore n(M \cap E) = n(M) + n(E) - n(M \cup E)$
$= 80 + 60 - 100 = 40$
Number of people whose speaks english only
$\because n(E - M) = n(E) - n(M \cap E)$
$= 60 - 40 = 20$

5. *(b)* Given, $a * b = \frac{a}{b} + \frac{b}{a} + \frac{1}{ab}$
$$\therefore 2*5 = \frac{2}{5} + \frac{5}{2} + \frac{1}{2 \times 5}$$
$$= \frac{2}{5} + \frac{5}{2} + \frac{1}{10} = \frac{4 + 25 + 1}{10} = \frac{30}{10} = 3$$

6. *(c)* Given, $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6\}$
$A - B = \{1, 3, 5\}$

7. *(b)* Given, $A = \{2, 3, 4, 5\}$, $B = \{36, 45, 49, 60, 77, 90\}$
R is the relation 'is factor from A to B.' $\{2, 3, 4, 5\}$ are not the factor of 49 and 77.
$\therefore$ Range of R is $\{36, 45, 60, 90\}$

8. *(c)* We have,
$$e^{(3+4i)x} = e^{3x}.e^{4ix}$$
$$= e^{3x}(\cos 4x + i \sin 4x)$$
$\therefore$ Real part of $e^{(3+4i)x}$ is $e^{3x} \cos 4x$

9. *(a)* We have,
$$z = x - iy \text{ and } (z)^{1/3} = p + iq$$
$\therefore (x - iy)^{1/3} = (p + iq)$
$$x - iy = (p + iq)^3$$
$$x - iy = p^3 + 3p^2qi + 3pq^2i^2 + i^3q^3$$
$$x - iy = p^3 + 3p^2qi - 3pq^2 - q^3i$$
$$x - iy = p^3 - 3pq^2 + (3p^2q - q^3)i$$
$\therefore x = p^3 - 3pq^2$ and $y = q^3 - 3p^2q$
$$\frac{x}{p} = p^2 - 3q^2 \text{ and } \frac{y}{q} = q^2 - 3p^2$$
$$\frac{x}{p} + \frac{y}{q} = p^2 - 3q^2 + q^2 - 3p^2 = -2(p^2 + q^2)$$
$$\frac{1}{p^2 + q^2}\left(\frac{x}{p} + \frac{y}{q}\right) = -2$$

10. *(a)* Given,
$$|z + i| = 2$$
$$|x + iy + i| = 2$$
$$\sqrt{x^2 + (y+1)^2} = 2$$
$$x^2 + (y+1)^2 = (2)^2$$
$\therefore$ Centre of circle is $(0, -1)$ and radius of circle is 2.

11. *(d)* Given, $2 + i$ is a root of $x^2 - 4x + c = 0$
If $2 + i$ is root of $x^2 - 4x + c = 0$
Then, $2 - i$ is also root of $x^2 - 4x + c = 0$
$\therefore (2 + i)(2 - i) = c$
$4 + 1 = c \Rightarrow c = 5$

12. *(c)* Given, $|z_1| = |z_2| = 2$ and $|z_1 + z_2| = 3$
Now, $\left|\frac{1}{z_1} + \frac{1}{z_2}\right| = \left|\frac{z_1 + z_2}{z_1 z_2}\right| = \frac{|z_1 + z_2|}{|z_1||z_2|} = \frac{3}{2 \times 2} = \frac{3}{4}$

13. *(d)* Given, $z = \frac{1 + \sin \pi - i \cos \pi}{1 + \sin \pi + i \cos \pi}$
$$\Rightarrow z = \frac{1 + 0 + i}{1 + 0 - i} = \frac{1 + i}{1 - i}$$
$$z = \left(\frac{1 + i}{1 - i}\right)\left(\frac{1 + i}{1 + i}\right) = \frac{1 - 1 + 2i}{1 + 1} = \frac{2i}{2} = i$$
Principal argument of $z = i$ is $\pi/2$.

14. *(c)* We have, $z_1 = 2 + 3i$, $z_2 = 3 + 2i$
$\therefore |z_1 + z_2| = |2 + 3i + 3 + 2i| = |5 + 5i|$
$|z_1 + z_2| = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$

15. *(d)* We have,

$$\frac{10i}{1+2i}=\frac{(10i)(1-2i)}{(1+2i)(1-2i)}=\frac{10(i+2)}{1+4}=\frac{10(2+i)}{5}$$

$$=2(2+i)=4+2i$$

16. *(e)* Let

$$S_n=\sum_{k=1}^{n}(3k^2+2k-1)$$

$$S_n=3\sum_{k=1}^{n}k^2+2\sum_{k=1}^{n}k-\sum_{k=1}^{n}1$$

$$S_n=\frac{3(n)(n+1)(2n+1)}{6}+\frac{2(n)(n+1)}{2}-n$$

Put $n=10$

$$\therefore\quad S_{10}=\frac{3(10)(11)(21)}{6}+\frac{2(10)(11)}{2}-10$$

$$=1155+110-10=1255$$

17. *(c)* Given, $a_1, a_2, a_3, \ldots, a_n$ are in AP.

and a_1, a_2, a_6 are in GP.

Let d is common difference of AP.

$\therefore\ a_2=a_1+d, a_6=a_2+5d$

Now, a_1, a_2, a_6 are in GP

$$\therefore\quad (a_2)^2=(a_1a_6)$$

$$(a_1+d)^2=(a_1)(a_1+5d)$$

$$a_1^2+2a_1d+d^2=a_1^2+5a_1d$$

$$2a_1+d=5a_1$$

$$d=5a_1-2a_1=3a_1$$

18. *(e)* Let $a_1, a_2, a_3, \ldots, a_n$ are in AP.

Given, $a_1+a_3=6$ … (i)

and $a_2+a_4=20$ … (ii)

$\therefore\ a_1+a_1+2d=6$

$2a_1+2d=6\Rightarrow a_1+d=3$ … (iii)

Subtract Eqs. (i) from (ii), we get

$$(a_2-a_1)+(a_4-a_3)=20-6$$

$$d+d=14\ \Rightarrow\ 2d=14$$

$$d=7$$

Putting the value of d in Eq. (iii), we get $a_1=-4$

$\therefore\ a_{11}=a_1+10d=-4+70=66$

19. *(a)* Given, in an AP.

$a_1=3$ and $a_n=17$

$S_n=70$

$$\therefore\quad S_n=\frac{n}{2}(a_1+a_n)$$

$$70=\frac{n}{2}(3+17)$$

$$n=\frac{70\times 2}{20}=7$$

$\therefore$ Number of terms in AP is 7.

20. *(d)* The positive rational number that are less than 1 and that have denominators as 30 in lowest terms are

$$\frac{1}{30},\frac{7}{30},\frac{11}{30},\frac{13}{30},\frac{17}{30},\frac{19}{30},\frac{23}{30},\frac{29}{30}$$

Their sum is $\dfrac{1+7+11+13+17+19+23+29}{30}$

$$=\frac{120}{30}=4$$

21. *(d)* Given, $p, q, 23$ is an increasing AP.

Such that p and q are prime numbers

The possible value of p and q are 11 and 17

$\therefore$ 11, 17, 23 are prime numbers and also in AP.

$\therefore\ p+q=11+17=28$

22. *(c)* Given, 5th and 7th terms of GP are 12 and 48

$\therefore\ a.r^4=12$ and $ar^6=48$

$$\frac{ar^6}{ar^4}=\frac{48}{12}\Rightarrow r^2=4\Rightarrow r=2$$

$$a(2)^4=12\Rightarrow a=\frac{12}{16}\Rightarrow a=\frac{3}{4}$$

$$\therefore\quad a_9=ar^8=\frac{3}{4}\times(2)^8=\frac{3}{4}\times 256=192$$

23. *(a)* The positive integer less than 1000 having only odd digits

One-digit odd number = 5

Two-digit odd numbers = $5^2=25$

Three-digit only odd numbers = $5^3=125$

$\therefore$ Total positive integer less than 1000 having only odd digits number is $5+25+125=155$.

24. *(d)* Number of polygons of three or more sides from 5 points is ${}^5C_3+{}^5C_4+{}^5C_5$

$$=10+5+1=16$$

25. *(e)* The middle terms of the expansion $\left(1+\frac{1}{5}\right)^{20}$ is ${}^{20}C_{10}\left(\frac{1}{5}\right)^{10}$.

26. *(c)* We have,

$${}^{11}C_0+{}^{11}C_1+{}^{11}C_2+{}^{11}C_3+{}^{11}C_4+{}^{11}C_5$$

$$=\frac{1}{2}({}^{11}C_0+{}^{11}C_0+{}^{11}C_1+{}^{11}C_1+{}^{11}C_2+{}^{11}C_2$$

$$+{}^{11}C_3+{}^{11}C_3+{}^{11}C_4+{}^{11}C_4+{}^{11}C_5+{}^{11}C_5)$$

$= \frac{1}{2}[{}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + {}^{11}C_3 + {}^{11}C_4 + {}^{11}C_5 + {}^{11}C_6 + {}^{11}C_7 + {}^{11}C_8 + {}^{11}C_9 + {}^{11}C_{10} + {}^{11}C_{11}]$ $[\because {}^nC_r = {}^nC_{n-r}]$

$= \frac{1}{2} \times 2^{11}$ $[\because {}^nC_0 + {}^nC_1 + ... + {}^nC_n = 2^n] = 2^{10}$

27. *(b)* We have, ${}^nP_r = 840$, ${}^nC_r = 35$

$\therefore \frac{n!}{(n-r)!} = 840$ and $\frac{n!}{r!(n-r)!} = 35$

$35 \times r! = 840$

$r! = \frac{840}{35} = 24$

$r! = 4! \Rightarrow r = 4$

28. *(a)* We have, $(1 + 2x - x^2)^{20}$

For sum of the coefficient of the expansion put $x = 1$, we get

$$(1 + 2 - 1)^{20} = 2^{20}$$

$\therefore$ The sum of coefficient $= 2^{20}$

29. *(c)* Number of ways of a committee of 4 people can be chosen from a panel of 10 people is

$${}^{10}C_4 = \frac{10!}{4!\,6!} = \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} = 210$$

30. *(d)* Given, $A = \begin{bmatrix} 6 & 2 \\ 7 & -5 \end{bmatrix}$

$$A - B = \begin{bmatrix} -2 & 1 \\ 4 & -9 \end{bmatrix}$$

$$\therefore \quad B = A - \begin{bmatrix} -2 & 1 \\ 4 & -9 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 2 \\ 7 & -5 \end{bmatrix} - \begin{bmatrix} -2 & 1 \\ 4 & -9 \end{bmatrix}$$

$$B = \begin{bmatrix} 8 & 1 \\ 3 & 4 \end{bmatrix}$$

31. *(e)* We have,

$$\begin{vmatrix} bc & ca & ab \\ a^3 & b^3 & c^3 \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \end{vmatrix}$$

Apply $C_1 \to aC_1, C_2 \to bC_2, C_3 = CC_3$

$$\therefore \quad \frac{1}{abc}\begin{vmatrix} abc & abc & abc \\ a^4 & b^4 & c^4 \\ 1 & 1 & 1 \end{vmatrix}$$

Taking common abc from R_1, we get

$$\begin{vmatrix} 1 & 1 & 1 \\ a^4 & b^4 & c^4 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

32. *(c)* $\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & k \\ -4 & 2 & 6 \end{bmatrix}$ is singular matrix

$$\therefore \begin{vmatrix} 1 & 2 & -1 \\ -3 & 4 & k \\ -4 & 2 & 6 \end{vmatrix} = 0$$

$1(24 - 2k) - 2(-18 + 4k) - 1(-6 + 16) = 0$

$24 - 2k + 36 - 8k - 10 = 0$

$-10k + 50 = 0$

$k = 5$

33. *(b)* We have,

$$\begin{bmatrix} -1 & 3 \\ 4 & -5 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 19 \\ \alpha & -27 \\ 0 & 14 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 19 \\ 4 & -27 \\ 0 & 14 \end{bmatrix} = \begin{bmatrix} -1 & 19 \\ \alpha & -27 \\ 0 & 14 \end{bmatrix}$$

$$\therefore \quad \alpha = 4$$

34. *(d)* We have, $A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 \\ 5 & -3 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} -\frac{3}{11} & \frac{4}{11} \\ \frac{5}{11} & \frac{-3}{11} \end{bmatrix}$$

$$|A^{-1}| = \frac{9}{121} - \frac{20}{121} = \frac{-11}{121} = -\frac{1}{11}$$

$$\text{adj } A^{-1} = \begin{bmatrix} \frac{-3}{11} & \frac{-4}{11} \\ \frac{-5}{11} & \frac{-3}{11} \end{bmatrix}$$

$$A = \frac{1}{|A^{-1}|} \text{adj } A^{-1}$$

$$A = \frac{1}{\frac{-1}{11}}\begin{bmatrix} \frac{-3}{11} & \frac{-4}{11} \\ \frac{-5}{11} & \frac{-3}{11} \end{bmatrix} = -11\begin{bmatrix} \frac{-3}{11} & \frac{-4}{11} \\ \frac{-5}{11} & \frac{-3}{11} \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}$$

35. *(a)* Given, system of equations

$$x + y + 2z = 4$$
$$3x + 3y + 6z = 17$$
and
$$5x - 3y + 2z = 27$$

In matrix form, this system of equations can be written as $AX = B$

Here, $D = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 5 & -3 & 2 \end{vmatrix}$

$$= 1(6+18) - 1(6-30) + 2(-9-15)$$
$$= 24 - 1(-24) + 2(-24)$$
$$= 24 + 24 - 48$$
$$= 48 - 48 = 0$$

$$D_1 = \begin{vmatrix} 4 & 1 & 2 \\ 17 & 3 & 6 \\ 27 & -3 & 2 \end{vmatrix}$$

$$= 4(6+18) - 1(34-162) + 2(-51-81)$$
$$= 4(24) - 1(-128) + 2(-132)$$
$$= 96 + 128 - 132 = 92 \neq 0$$

$$D_2 = \begin{vmatrix} 1 & 4 & 2 \\ 3 & 17 & 6 \\ 5 & 27 & 2 \end{vmatrix}$$

$$= 1(34-162) - 4(6-30) + 2(81-85)$$
$$= 1(-128) - 4(-24) + 2(-4)$$
$$= -128 + 96 - 8 = -40$$

$$D_3 = \begin{vmatrix} 1 & 1 & 4 \\ 3 & 3 & 17 \\ 5 & 3 & 27 \end{vmatrix}$$

$$= 1(81-51) - 1(81-85) + 4(9-15)$$
$$= (30) - 1(-4) + 4(-6)$$
$$= 30 + 4 - 24 = 34 - 24 = 10$$

Here, $D = 0$ and atleast one of D_1, D_2 and D_3 is non-zero, then given system is inconsistent or no solution.

36. *(c)* Given, inequality

$$\frac{2n-3}{3} \geq \frac{n-1}{6} + 1$$

$$\Rightarrow \quad \frac{2n-3}{3} - \frac{n-1}{6} \geq 1$$

$$\Rightarrow \quad \frac{4n-6-n+1}{6} \geq 1 \quad \Rightarrow \frac{3n-5}{6} \geq 1$$

$$\Rightarrow \quad 3n \geq 11 \quad \Rightarrow n \geq \frac{11}{3} \Rightarrow n = 5$$

$\therefore$ Smallest prime number satisfying the given inequality is 5.

37. *(e)* We have, $|n^2 - 100| < 50$

$$\Rightarrow \quad -50 < n^2 - 100 < 50$$

$$\Rightarrow \quad 50 < n^2 < 150$$

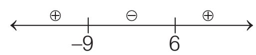
$n = -12, -11, -10, -9, -8, 8, 9, 10, 11, 12$

$\therefore$ Number of integers are 10.

38. *(d)* Given, inequality $\dfrac{x+9}{x-6} \leq 0$

Here, $x + 9 = 0 \Rightarrow x = -9$

and $x - 6 = 0 \Rightarrow x = 6$



$\therefore$ Required solution set is $[-9, 6)$.

39. *(b)* (i) False

(ii) True

(iii) Can not assign true or false so, it is not a statement.

40. *(b)* Given, $\sin^2\theta + 2\cos^2\theta = \dfrac{7}{4}$

$$\Rightarrow \quad 1 - \cos^2\theta + 2\cos^2\theta = \frac{7}{4}$$

$$\Rightarrow \quad 1 + \cos^2\theta = 7/4 \quad \Rightarrow \cos^2\theta = 3/4$$

$$\Rightarrow \quad \cos\theta = \sqrt{3}/2$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30° \; (0 \leq \theta < = 90°)$$

41. *(c)* $\sin^2 1° + \sin^2 2° + \sin^2 3° + \ldots + \sin^2 88° + \sin^2 89°$

$$= \sin^2 1° + \sin^2(90° - 1°) + \sin^2 2° + \sin^2(90 - 2°) + \sin^2 3° + \sin^2(90° - 3°) + \ldots + \sin^2 45°$$

$$= \sin^2 1° + \cos^2 1° + \sin^2 2° + \cos^2 2° + \sin^2 3° + \cos^2 3° + \ldots + \sin^2 45°$$

$$= (1 + 1 + 1 + \ldots 44 \text{ times}) + \left(\frac{1}{\sqrt{2}}\right)^2 = 44 + \frac{1}{2} = \frac{89}{2}$$

42. *(b)* $\sin^4 \dfrac{\pi}{8} + \sin^4 \dfrac{3\pi}{8}$

$$= \left(\frac{\sqrt{2-\sqrt{2}}}{2}\right)^4 + \left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)^4$$

$$= \left(\frac{2-\sqrt{2}}{4}\right)^2 + \left(\frac{2+\sqrt{2}}{4}\right)^2$$

$$= \frac{4+2-4\sqrt{2}}{16} + \frac{4+2+4\sqrt{2}}{16} = \frac{12}{16} = \frac{3}{4}$$

43. *(e)* $\sin(45° + \theta) - \cos(45° - \theta)$

$= \sin 45° \cos\theta + \sin\theta \cos 45° - \cos 45° \cos\theta - \sin 45° \sin\theta$

$= \frac{1}{\sqrt{2}}\cos\theta + \frac{\sin\theta}{\sqrt{2}} - \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta$

$= 0$

44. *(a)* $\cos 2x = \cos x$

$\Rightarrow 2\cos^2 x - 1 = \cos x$

$\Rightarrow 2\cos^2 x - \cos x - 1 = 0$

$\Rightarrow 2\cos^2 x - 2\cos x + \cos x - 1 = 0$

$\Rightarrow 2\cos x(\cos x - 1) + 1(\cos x - 1) = 0$

$\Rightarrow (\cos x - 1)(2\cos x + 1) = 0$

$\Rightarrow \cos x = 1$ or $2\cos x = -1$

$\Rightarrow x = 0$ or $\cos x = -\frac{1}{2}$

$\Rightarrow x = 0$ or $x = \frac{2\pi}{3}$

45. *(b)* $10\tan(\cos^{-1} 3 + \cot^{-1} 7)$

$= 10\tan\left(\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}\right)$

$= 10\tan\left[\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{3}\frac{1}{7}}\right)\right]$

$= 10\left(\frac{7+3}{21-1}\right) = 10\left(\frac{10}{20}\right) = 5$

46. *(e)* $\tan x + \tan y = \frac{5}{6}$ ... (i)

and $\cot x + \cot y = 5$

$\Rightarrow \frac{1}{\tan x} + \frac{1}{\tan y} = 5$

$\Rightarrow \frac{\tan x + \tan y}{\tan x \tan y} = 5$ ... (ii)

On dividing Eqs. (i) by (ii), we get

$\tan x \tan y = \frac{1}{6}$

$\therefore \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\Rightarrow \tan(x + y) = \frac{5/6}{1 - 1/6}$

$= \frac{5/6}{5/6} = 1$

47. *(a)* $\frac{\sin 91° + \sin 1°}{\sin 91° - \sin 1°}$

$= \frac{2\sin\left(\frac{91° + 1°}{2}\right)\cos\left(\frac{91° - 1°}{2}\right)}{2\cos\left(\frac{91° + 1°}{2}\right)\sin\left(\frac{91° - 1°}{2}\right)}$

$= \frac{2\sin(46°)\cos 45°}{2\cos(46°)\sin 45°} = \tan 46°$

48. *(d)* $\cos\left(\cos^{-1}\frac{1}{5} + 2\sin^{-1}\frac{1}{5}\right)$

$= \cos\left(\cos^{-1}\frac{1}{5} + \sin^{-1}\left(2\times\frac{1}{5}\sqrt{1-\frac{1}{25}}\right)\right)$

$= \cos\left(\cos^{-1}\frac{1}{5} + \sin^{-1}\left(\frac{2}{5}\sqrt{\frac{24}{25}}\right)\right)$

$= \cos\left(\cos^{-1}\frac{1}{5} + \sin^{-1}\left(\frac{4\sqrt{6}}{25}\right)\right)$

$= \cos\left(\cos^{-1}\frac{1}{5} + \cos^{-1}\left(\frac{23}{25}\right)\right)$

$= \cos\left(\cos^{-1}\left(\frac{1}{5}\times\frac{23}{25} - \sqrt{1-\frac{1}{25}}\sqrt{1-\frac{529}{625}}\right)\right)$

$= \cos\left(\cos^{-1}\left(\frac{23}{125}\right) - \sqrt{\frac{24}{25}}\sqrt{\frac{96}{625}}\right)$

$= \cos\left[\cos^{-1}\left(\frac{23}{125} - \frac{\sqrt{24}.2\sqrt{24}}{5\times 25}\right)\right]$

$= \cos\left[\cos^{-1}\left(\frac{23}{125} - \frac{2\times 24}{125}\right)\right]$

$= \frac{23-48}{125} = \frac{-25}{125} = -\frac{1}{5}$

49. *(b)* Equation of the line passing through the point (x_1, y_1) and whose slope i.e., m is

$y - y_1 = m(x - x_1)$

Here, $x_1 = -3, y_1 = 7$ and $m = 0$

$\therefore y - 7 = 0\,(x + 3) \Rightarrow y = 7$

50. *(e)* Given parabola

$y = ax^2 + 5x - 2$

It is passes through (1, 5)

$\therefore 5 = a + 5 - 2 \Rightarrow a = 2$

$y = mx + 2$ intersect the parabola at (1, 5)

$\therefore$ (1, 5) lies on line $y = mx + 2$

$5 = m + 2 \Rightarrow m = 3$

$\therefore a + m = 2 + 3 = 5$

51. *(e)* Points $P(7, 5)$, $Q(a, 2a)$, $R(12, 30)$ are collinear

$\therefore$ Slope of PQ = Slope of PR

$$\frac{2a-5}{a-7} = \frac{30-5}{12-7}$$

$$\frac{2a-5}{a-7} = \frac{25}{5} = 5$$

$$2a - 5 = 5a - 35$$

$$3a = 30 \Rightarrow a = 10$$

52. *(c)* The straight line $4x + 6y = 5$ and $6x + ky = 3$ are parallel

$$\therefore \quad \frac{4}{6} = \frac{6}{k} \Rightarrow k = \frac{36}{4} \Rightarrow k = 9$$

53. *(a)* We have,

$(a, 2)$ is the point of intersection of the straight line $y = 2x - 4$ and $y = x + c$

$\therefore$ a, 2 lie on both line

$$2 = 2a - 4 \Rightarrow a = 3 \Rightarrow 2 = a + c$$

$$\Rightarrow \quad 2 = 3 + c \Rightarrow c = -1$$

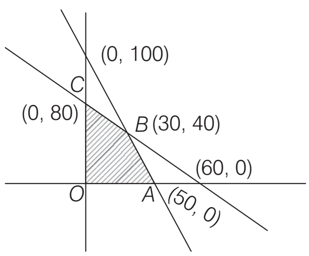
54. *(d)* Given,

Maximise $z = 7x + 5y$

Subject to constant

$2x + y \le 100$

$4x + 3y \le 240$ $x, y \ge 0$



Corner Point	$z = 7x + 5y$
O (0, 0)	0
A (50, 0)	350
B (30, 40)	210 + 200 = 410
C (0, 80)	400

Clearly the maximum value of $z = 7x + 5y$ is 410.

55. *(e)* Equation of circle passing through $(-1, 1)$ and whose centre $(3, 6)$ is

$$(x-3)^2 + (y-6)^2 = (-1-3)^2 + (1-6)^2$$

$$x^2 - 6x + 9 + y^2 - 12y + 36 = 16 + 25$$

$$x^2 + y^2 - 6x - 12y + 4 = 0$$

56. *(c)* Given, equation of circle

$$x^2 + y^2 - 4x + 2y = 0$$

$$(x-2)^2 + (y+1)^2 = (\sqrt{5})^2$$

$\therefore$ Centre $(2, -1)$ radius $= \sqrt{5}$

57. *(b)* Equation of circle whose radius is $\sqrt{7}$ and concentric with the circle

$x^2 + y^2 - 8x + 6y - 11 = 0$ centre of circle $(4, 3)$

$\therefore$ Equation of circle whose centre $(4, 3)$ and radius $\sqrt{7}$ is

$$(x-4)^2 + (y+3)^2 = (\sqrt{7})^2$$

$$x^2 - 8x + 16 + y^2 + 6y + 9 = 7$$

$$x^2 + y^2 - 8x + 6y + 18 = 0$$

58. *(d)* Given parabola

$$y = x^2 - 2x + 4$$

$$(x-1)^2 = y - 3$$

Vertex $(1, 3)$

When vertex shifted p units to right and then q unit up

$$\therefore \quad (1 + p, 3 + q) \equiv (4, 5)$$

$$\therefore \quad p = 3, q = 2$$

59. *(b)* Given parabola

$$y = (x-2)(x-8) + 7$$

$$y = x^2 - 10x + 16 + 7$$

$$(x-5)^2 = y + 2$$

vertex $(5, -2)$

60. *(c)* Given ellipse

$$400x^2 + 100y^2 = 40000$$

$$\frac{x^2}{100} + \frac{y^2}{400} = 1$$

Here, $a^2 = 100$, $b^2 = 400$

$$\therefore \quad a = 10, \quad b = 20$$

The length of major axis $= 2b = 2 \times 20 = 40$

and length of minor axis $= 2a = 2 \times 10 = 20$

61. *(d)* We have, equation of ellipse $\frac{x^2}{1} + \frac{y^2}{4} = 1$

On comparing it with $\frac{x^2}{a^2} + \frac{y^2}{b^2}$, we get

$a = 1$ and $b = 2$

$$\therefore \text{Eccentricity, } e = \sqrt{1 - \frac{a^2}{b^2}} \qquad (a < b)$$

$$= \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

62. *(e)* We have, equation of hyperbola

$$3x^2 - 2y^2 = 6 \Rightarrow \frac{x^2}{2} - \frac{y^2}{3} = 1$$

On comparing it with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get $a = \sqrt{2}$, $b = \sqrt{3}$

$\therefore$ Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times (\sqrt{3})^2}{\sqrt{2}} = 3\sqrt{2}$

63. *(b)* We have,

$$\mathbf{u} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}} \text{ and } \mathbf{v} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$

$\therefore |\mathbf{u}| = \sqrt{(1)^2 + (-3)^2 + (2)^2} = \sqrt{1 + 9 + 4} = \sqrt{14}$

and $|\mathbf{v}| = \sqrt{(2)^2 + (4)^2 + (-5)^2} = \sqrt{4 + 16 + 25} = \sqrt{45}$

Now, we know that

$$|\mathbf{u} \times \mathbf{v}|^2 + |\mathbf{u}.\mathbf{v}|^2 = |\mathbf{u}|^2 |\mathbf{v}|^2$$
$$= (\sqrt{14})^2 (\sqrt{45})^2$$
$$= 14 \times 45 = 630$$

64. *(b)* Let $\mathbf{a} = \hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

$\therefore \quad |\mathbf{a}| = \sqrt{(1)^2 + (-5)^2 + (8)^2}$

$= \sqrt{1 + 25 + 64}$

$= \sqrt{90} = 3\sqrt{10}$

$\therefore \quad \mathbf{a} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{1}{3\sqrt{10}}\hat{\mathbf{i}} - \frac{5}{3\sqrt{10}}\hat{\mathbf{j}} + \frac{8}{3\sqrt{10}}\hat{\mathbf{k}}$

$\therefore$ d'c of $\mathbf{a} = \left(\frac{1}{3\sqrt{10}}, -\frac{5}{3\sqrt{10}}, \frac{8}{3\sqrt{10}}\right)$

65. *(b)* We have,

$$\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}, \mathbf{b} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$\therefore \cos\theta = \frac{\mathbf{a} . \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|}$

$= \frac{(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) . (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}})}{\sqrt{(1)^2 + (1)^2 + (-1)^2}\sqrt{(2)^2 + (3)^2 + (1)^2}}$

$= \frac{2 + 3 - 1}{\sqrt{3} \times \sqrt{14}} = \frac{4}{\sqrt{42}}$

Now, $\sin\theta = \sqrt{1 - \cos^2\theta}$

$= \sqrt{1 - \frac{16}{42}} = \sqrt{\frac{26}{42}}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\sqrt{\frac{26}{42}}\right)}{\left(\frac{4}{\sqrt{42}}\right)} = \frac{\sqrt{26}}{4}$

66. *(d)* Let

$$\mathbf{a} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}} \text{ and } \mathbf{b} = 3\hat{\mathbf{i}} + 2\lambda\hat{\mathbf{j}}$$

Since, $\mathbf{a}$ and $\mathbf{b}$ are perpendicular

$\therefore \quad \mathbf{a} . \mathbf{b} = 0$

$\Rightarrow (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) . (3\hat{\mathbf{i}} + 2\lambda\hat{\mathbf{j}}) = 0$

$\Rightarrow \quad 6 - 2\lambda + 0 = 0 \Rightarrow \lambda = 3$

67. *(e)* We have,

$|\alpha\hat{\mathbf{i}} + (\alpha + 1)\hat{\mathbf{j}} + 2\hat{\mathbf{k}}| = 3$

$\Rightarrow \sqrt{\alpha^2 + (\alpha + 1)^2 + 4} = 3$

$\Rightarrow \alpha^2 + (\alpha + 1)^2 + 4 = 9$

$\Rightarrow \alpha^2 + \alpha^2 + 2\alpha + 1 + 4 - 9 = 0$

$\Rightarrow 2\alpha^2 + 2\alpha - 4 = 0$

$\Rightarrow \alpha^2 + \alpha - 2 = 0$

$\Rightarrow (\alpha + 2)(\alpha - 1) = 0$

$\Rightarrow \alpha = -2, 1$

68. *(d)* We have,

$$\mathbf{a} = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} \text{ and } \mathbf{b} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$\therefore \mathbf{a} + \mathbf{b} = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$= 4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

and $\mathbf{a} - \mathbf{b} = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$

$= 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

Now, $(\mathbf{a} + \mathbf{b}) . (\mathbf{a} - \mathbf{b}) = (4\hat{\mathbf{i}} + \hat{\mathbf{j}} + 4\hat{\mathbf{k}}) . (3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$

$= 0 + 3 + 8 = 11$

69. *(e)* We have,

$$\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}} \text{ and } \mathbf{b} = \lambda\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

Now, we have given

projection of $\mathbf{a}$ and $\mathbf{b}$ = projection of $\mathbf{b}$ on $\mathbf{a}$

$\Rightarrow \quad \frac{\mathbf{a} . \mathbf{b}}{|\mathbf{b}|} = \frac{\mathbf{a} . \mathbf{b}}{|\mathbf{a}|} \quad \Rightarrow |\mathbf{a}| = |\mathbf{b}|$

$\Rightarrow \sqrt{(1)^2 + (2)^2 + (-3)^2} = \sqrt{(0)^2 + (\lambda)^2 + (3)^2}$

$\Rightarrow \quad 1 + 4 + 9 = \lambda^2 + 9$

$\Rightarrow \quad \lambda^2 = 5 \Rightarrow \lambda = \pm\sqrt{5}$

70. *(a)* We have,

$$|\mathbf{a}| = 2, |\mathbf{b}| = 3, \mathbf{a} . \mathbf{b} = 4$$

Now, we know that

$$|\mathbf{a} - \mathbf{b}|^2 = (\mathbf{a} - \mathbf{b}) . (\mathbf{a} - \mathbf{b})$$
$$= |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a} . \mathbf{b}$$
$$= (2)^2 + (3)^2 - 2(4) = 4 + 9 - 8 = 5$$

$\therefore \quad |\mathbf{a} - \mathbf{b}| = \sqrt{5} \quad [\because |\mathbf{x}| \geq 0]$

71. *(d)* We have,

$$\frac{x-1}{2}=\frac{y+1}{4}=\frac{z-2}{-2}$$

On putting $x=3$, $y=3$, $z=0$, we get

$$\Rightarrow \quad \frac{3-1}{2}=\frac{3+1}{4}=\frac{0-2}{-2} \Rightarrow \frac{2}{2}=\frac{4}{4}=\frac{-2}{-2}$$

$$\Rightarrow \quad 1=1=1$$

Which is true

$\therefore$ (3, 3, 0) lies on given line.

72. *(b)* We have,

$$\mathbf{a}=\hat{\mathbf{j}}+\hat{\mathbf{k}} \text{ and } \mathbf{n}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$$

$\therefore$ Equation of plane will be

$$\mathbf{r}\,.\,\mathbf{n}=\mathbf{a}\,.\,\mathbf{n}$$

$$\Rightarrow \quad \mathbf{r}\,.\,(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})=(\hat{\mathbf{j}}+\hat{\mathbf{k}})\,.\,(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})$$

$$\Rightarrow \quad \mathbf{r}\,.\,(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})=1+1$$

$$\Rightarrow \quad \mathbf{r}\,.\,(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})=2$$

$$\Rightarrow (x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})\,.\,(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}})=2$$

$$[\because \mathbf{r}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}]$$

$$\Rightarrow \quad x+y+z=2$$

Which the equation of required plane.

73. *(c)* We have, equation of plane

$$\mathbf{r}\,.\,(6\hat{\mathbf{i}}+2\hat{\mathbf{j}}-9\hat{\mathbf{k}})=46$$

or $\mathbf{r}\,.\,(6\hat{\mathbf{i}}+2\hat{\mathbf{j}}-9\hat{\mathbf{k}})-46=0$

$$6x+2y-9z-46=0$$

Now, the distance of any point (x_1, y_1, z_1) from the plane $ax+by+cz+d=0$ is given by

$$\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$\therefore \text{ Required distance}=\left|\frac{6\times 4+2\times 2-9\times 3-46}{\sqrt{(6)^2+(2)^2+(-9)^2}}\right|$$

$$=\left|\frac{24+4-27-46}{\sqrt{36+4+81}}\right|$$

$$=\left|\frac{-45}{\sqrt{121}}\right|=\frac{45}{11}$$

74. *(c)* We have, equation of plane is

$$\mathbf{r}\,.\,(3\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})=18$$

$$\Rightarrow (x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}})\,.\,(3\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})=18$$

$$[\because \mathbf{r}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}]$$

$$\Rightarrow \quad 3x+y+2z=18$$

$$\Rightarrow \quad \frac{3x}{18}+\frac{y}{18}+\frac{2z}{18}=\frac{18}{18}$$

$$\Rightarrow \quad \frac{x}{6}+\frac{y}{18}+\frac{z}{9}=1$$

$\therefore$ Intercept on coordinate axes = 6, 18, 9

$\therefore$ Required sum = 6 + 18 + 9 = 33

75. *(a)* We have, equation of line

$$\frac{x-2}{1}=\frac{y-4}{-5}=\frac{z+3}{4}=\lambda \quad \dots \text{(i)}$$

and equation of *xy*-plane

$$z=0$$

Any point on line (i), is given by

$$(\lambda+2, -5\lambda+4, 4\lambda-3)$$

The above point also lie on the plane $z=0$

$$\therefore \quad 4\lambda-3=0$$

$$\Rightarrow \quad \lambda=\frac{3}{4}$$

$$\therefore \text{ Required point}=\left(\frac{3}{4}+2, -5\times\frac{3}{4}+4, 4\times\frac{3}{4}-3\right)$$

$$=\left(\frac{11}{4}, \frac{1}{4}, 0\right)$$

76. *(d)* We know that any line passing through the point (1, –1, 2) and (7, 0, 5) is given by

$$\frac{x-1}{7-1}=\frac{y-(-1)}{0-(-1)}=\frac{z-2}{5-2}$$

$$\Rightarrow \quad \frac{x-1}{6}=\frac{y+1}{1}=\frac{z-2}{3}$$

77. *(a)* We have, equation of planes

$$x+y+z=1 \text{ and } x-2y+3z=1$$

Let θ be angle between these planes, then

$$\cos\theta=\frac{(1)(1)+(1)(-2)+(1)(3)}{\sqrt{(1)^2+(1)^2+(1)^2}\sqrt{(1)^2+(-2)^2+(3)^2}}$$

$$=\frac{1-2+3}{\sqrt{3}\times\sqrt{14}}=\frac{2}{\sqrt{42}}$$

$$\therefore \quad \theta=\cos^{-1}\left(\frac{2}{\sqrt{42}}\right)$$

78. *(e)* We have,

The equation of a plane passing through the intersection of the planes $x+2y-z=3$ and $x+y-3z=5$ is

$$(x+2y-z-3)+\lambda\,(x+y-3z-5)=0$$

Also, this plane passes through (1, –1, 0)

$$\therefore \quad (1-2-3)+\lambda(1-1-5)=0$$

$$\Rightarrow \quad -4-5\lambda=0$$

$$\Rightarrow \quad \lambda=-\frac{4}{5}$$

$\therefore$ Equation of required plane is

$$x + 2y - z - 3 + \left(-\frac{4}{5}\right)(x + y - 3z - 5) = 0$$

$\Rightarrow 5x + 10y - 5z - 15 - 4x - 4y + 12z + 20 = 0$

$\Rightarrow \quad x + 6y + 7z + 5 = 0$

79. *(d)* Let the mean marks of two student left out be x.

Now, according to the question

$$28 \times 82 + 2 \times x = 30 \times 80$$

$\Rightarrow \quad 2296 + 2x = 2400$

$\Rightarrow \quad 2x = 104 \quad \Rightarrow \quad x = 52$

$\therefore$ Mean marks of the two left out students is 52.

80. *(a)* We have, two dice are rolled

$\therefore$ Total number of out comes, $n(s) = 6 \times 6 = 36$

Now, let E be the event that sum of the numbers on the top faces being a prime number.

$\therefore E = \{(2, 3), (3, 2), (2, 5), (5, 2), (11, 2), (2, 11)\}$

$\Rightarrow n(E) = 6$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

81. *(e)* Total number of ways of choosing 3 numbers from $\{1, 2, 3, 4, 5\} = {}^5C_3 = \dfrac{5 \times 4}{2 \times 1} = 10$

$\therefore \quad n(S) = 10$

Let E be the event of selecting three numbers which in AP.

$\therefore \quad E = \{(1, 2, 3), (2, 3, 4), (3, 4, 5), (1, 3, 5)\}$

$\Rightarrow n(E) = 4$

$\therefore$ Required probability $= \dfrac{n(E)}{n(S)} = \dfrac{4}{10} = \dfrac{2}{5}$

82. *(c)* Mean score

$$= \frac{20 \times 70 + 40 \times 80 + 30 \times 90 + 10 \times 100}{100}$$

$$= \frac{1400 + 3200 + 2700 + 1000}{100}$$

$= 14 + 32 + 27 + 10$

$= 83$

83. *(b)* We have, A and B are mutually exclusive events

$\therefore \quad n(A \cap B) = \phi$

$\Rightarrow \quad P(A \cap B) = 0$

Now, we know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\Rightarrow \quad 0.75 = 0.5 + P(B) - 0$

$\Rightarrow \quad P(B) = 0.75 - 0.5 = 0.25$

84. *(b)* The total number of balls $= 7 + 6 + 4 + 3 = 20$

$\therefore \quad P(\text{Red}) = \dfrac{\text{Number of Red ball}}{\text{Total Number of Ball}} = \dfrac{3}{20}$

85. *(e)* We have,

$$\Sigma P(x) = 1$$

$\Rightarrow a + 2a + 3a + 4a + 5a = 1$

$\Rightarrow \quad 15a = 1$

$\Rightarrow \quad a = \dfrac{1}{15}$

Now, $E(X) = \Sigma XP(X)$

$= (-1)(a) + (0)(2a) + (1)(3a) + (2)(4a) + (3)(5a)$

$= -a + 0 + 3a + 8a + 15a = 25a$

$= 25 \times \dfrac{1}{15} = \dfrac{5}{3}$

86. *(b)* We have,

$$f(x) = \begin{cases} 1 - 5x, & x < -2 \\ x^2 - 2x, & -2 \le x \le 1 \\ -1 + 2x, & x > 1 \end{cases}$$

$\therefore \quad f(-1) = (-1)^2 - 2(-1)$

$[\because -1 \in [-2, 1], \text{ so } f(x) = x^2 - 2x]$

$= 1 + 2 = 3$

87. *(a)* We have,

$$\frac{dy}{dx} = \frac{2x - y}{x + 2y}$$

This is homogeneous differential equation.

So, put $y = vx$ and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = \frac{2x - vx}{x + 2vx}$$

$\Rightarrow \quad x\dfrac{dv}{dx} = \dfrac{2 - v}{1 + 2v} - v$

$\Rightarrow \quad x\dfrac{dv}{dx} = \dfrac{2 - v - v - 2v^2}{1 + 2v}$

$\Rightarrow \quad x\dfrac{dv}{dx} = -\left(\dfrac{2v^2 + 2v - 2}{1 + 2v}\right)$

$\Rightarrow \quad x\dfrac{dv}{dx} = -2\left(\dfrac{v^2 + v - 1}{1 + 2v}\right)$

$\Rightarrow \quad \dfrac{1 + 2v}{v^2 + v - 1}\, dv = \dfrac{-2}{x} dx$

On integrating both the sides, we get

$$\int \frac{1 + 2v}{v^2 + v - 1}\, dv = -2\int \frac{1}{x} dx$$

$\Rightarrow \quad \log(v^2 + v - 1) = -2\log x + \log c'$

$$\Rightarrow \log(v^2 + v - 1) = \log\left(\frac{c'}{x^2}\right)$$

$$\Rightarrow v^2 + v - 1 = \frac{c'}{x^2}$$

$$\Rightarrow \frac{y^2}{x^2} + \frac{y}{x} - 1 = \frac{c'}{x^2}$$

$$\Rightarrow y^2 + xy - x^2 = c'$$

$$\Rightarrow x^2 - y^2 - xy = -c'$$

$\Rightarrow x^2 - y^2 - xy = c$, where $c = -c'$

which is required solution.

88. *(e)* We have,

$$\lim_{x \to 3} \frac{e^{x-3} - x + 1}{x^2 - \log(x-2)} = \frac{e^{3-3} - 3 + 1}{(3)^2 - \log(3-2)}$$

$$= \frac{e^\circ - 2}{9 - \log 1} = \frac{1-2}{9-0} = \frac{-1}{9}$$

89. *(e)* We have,

$$\lim_{x \to 4} \frac{\sqrt{x^2+9} - 5}{x - 4} \left(\frac{0}{0}\right)$$

On applying L', Hospital Rule, we get

$$\lim_{x \to 4} \frac{\frac{1}{2\sqrt{x^2+9}}(2x) - 0}{1 - 0} = \lim_{x \to 4} \frac{x}{\sqrt{x^2+9}} = \frac{4}{5}$$

90. *(d)* We have,

$$f(x) = \begin{cases} cx^2 + 2x, & \text{if } x < 2 \\ 2x + 4, & \text{if } x \geq 2 \end{cases}$$

Since $f(x)$ is continuous on $(-\infty, \infty)$, then it will be continuous at $x = 2$

$\therefore$ LHL (at $x = 2$) = RHL (at $x = 2$) = $f(2)$

$$\Rightarrow \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\Rightarrow \lim_{x \to 2^-} (cx^2 + 2x) = \lim_{x \to 2^+} (2x + 4)$$

$$\Rightarrow \lim_{h \to 0} c(2-h)^2 + 2(2-h) = \lim_{h \to 0} 2(2+h) + 4$$

$$\Rightarrow 4c + 4 = 4 + 4 \Rightarrow 4c = 4 \Rightarrow c = 1$$

91. *(a)* We have,

$$\lim_{x \to 0} \frac{x^{100} \sin 7x}{(\sin x)^{101}}$$

$$= \lim_{x \to 0} x^{100} \left(\frac{\sin 7x}{7x}\right) \times \left(\frac{1}{\frac{\sin^{101} x}{x^{101}}}\right) \times \frac{7x}{x^{101}}$$

$$= \lim_{x \to 0} 7 \left(\frac{\sin 7x}{7x}\right) \times \left(\frac{1}{\frac{\sin^{101} x}{x^{101}}}\right)$$

$$= 7 \times 1 \times \frac{1}{1} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 7$$

92. *(b)* We have,

$$f(x) = \frac{5}{2}x^2 - e^x$$

$$\therefore f'(x) = \frac{5}{2}(2x) - e^x = 5x - e^x$$

and $f''(x) = 5 - e^x$

Now, we have $f''(c) = 0$

$$\Rightarrow 5 - e^c = 0$$

$$\Rightarrow e^c = 5$$

$$\Rightarrow c = \log 5$$

93. *(e)* We have,

$$y = (\cos x)^{2x}$$

$$\therefore \log y = 2x \log \cos x$$

On differentiating both the sides, we get

$$\frac{1}{y}\frac{dy}{dx} = 2\log \cos x + \frac{2x}{\cos x}(-\sin x)$$

$$\Rightarrow \frac{dy}{dx} = y[2\log \cos x - 2x \tan x]$$

$$\Rightarrow \frac{dy}{dx} = 2(\cos x)^{2x}(\log\cos x - x \tan x)$$

94. *(a)* We have,

$$x^3 + 2xy + \frac{1}{3}y^3 = \frac{11}{3}$$

On differentiating both the sides with respect to x, we get

$$3x^2 + 2\left[y + x\frac{dy}{dx}\right] + \frac{1}{3}\left(3y^2\frac{dy}{dx}\right) = 0$$

$$\Rightarrow 3x^2 + 2y + 2x\frac{dy}{dx} + y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(2x + y^2) = -(3x^2 + 2y)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(3x^2 + 2y)}{2x + y^2}$$

$$\therefore \left.\frac{dy}{dx}\right|_{(2,-1)} = -\frac{(3(2)^2 + 2(-1))}{2(2) + (-1)^2}$$

$$= \frac{-[12-2]}{4+1} = -\frac{10}{5} = -2$$

95. *(c)* We have,

$$f(x) = \begin{cases} x^2, & \text{for} \quad x \le 1 \\ 1, & \text{for} \quad 1 < x \le 3 \\ 5 - 2x, & \text{for} \quad x > 3 \end{cases}$$

Now, it is clear that, at $x = 6$

$f(x) = 5 - 2x$ $\quad [\because f(x) = 5 - 2x, \text{ for } x > 3]$

$\therefore \; f'(x) = -2$

$\Rightarrow \; f'(6) = -2$

96. *(e)* We have,

$F(x) = (f(g(x)))^2$

$\therefore \quad F'(x) = 2f(g(x)) \,.\, f'(g(x)).g'(x)$

$\Rightarrow \quad F'(1) = 2f(g(1)).f'(g(1)).g'(1)$

$= 2f(2) \,.\, f'(2).(3) \quad [\because g(1) = 2, g'(1) = 3]$

$= 2 \times 4 \times 5 \times 3 \quad [\because f(2) = 4, f'(2) = 5]$

$= 120$

97. *(c)* We have,

$y = 2 + \sqrt{u}$ and $u = x^3 + 1$

$\Rightarrow \quad y = 2 + \sqrt{x^3 + 1}$

On differentiating both the sides with respect to x, we get

$$\frac{dy}{dx} = 0 + \frac{1}{2\sqrt{x^3+1}} \,.\, 3x^2 = \frac{3x^2}{2\sqrt{x^3+1}}$$

98. *(b)* The equation of curve is

$y = -2x^2 + 3$

$\therefore \quad \frac{dy}{dx} = -4x$

Now, slope of tangent at $x = 1$

$= \left.\frac{dy}{dx}\right|_{x=1} = -4(1)$

$\therefore \quad m = -4$

Again, when $x = 1$

$\therefore \; y = -2(1)^2 + 3 = -2 + 3 = 1$

$\therefore$ Equation of tangent is given by

$y - 1 = -4(x - 1)$ $\quad [\because$Equation $]$

$\Rightarrow \quad y - 1 = -4x + 4$

$\Rightarrow \quad y = -4x + 5$

99. *(c)* We have,

$f(x) = x^3 e^x$

$\therefore \; f'(x) = 3x^2 e^x + x^3 e^x$

$= x^2 e^x (3 + x)$

For increasing, $\quad f'(x) > 0$

$\Rightarrow \quad x^2 e^x (3 + x) > 0$

$\Rightarrow \quad f'(x) > 0$ for $x > -3$

$\therefore \; f(x)$ is increasing for $x > -3$ or $x \in (-3, \infty)$.

100. *(b)* By Langrange's Mean value theorem, we have

$$f'(c) = \frac{f(b) - f(a)}{b - a}, \; c \in (4, 16)$$

$$\Rightarrow \quad \frac{1}{2\sqrt{c}} = \frac{4 - 2}{16 - 4}$$

$\left[\because f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}}\right.$

and $f_{(4)} = 2$ and $\left. f_{(16)} = 4\right]$

$$\Rightarrow \quad \frac{1}{2\sqrt{c}} = \frac{2}{12}$$

$\Rightarrow \quad \sqrt{c} = 3 \;\Rightarrow\; c = 9 \in (4, 16)$

$\therefore \quad c = 9$

101. *(e)* We have, $f(x) = (x^2 - 3)e^x$

$\therefore \quad f'(x) = 2xe^x + (x^2 - 3)e^x$

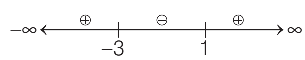
$= e^x (x^2 + 2x - 3)$

$= e^x (x + 3)(x - 1)$

For decreasing, $f'(x) < 0$

$\Rightarrow \quad e^x (x + 3)(x - 1) < 0$

$\Rightarrow \quad (x + 3)(x - 1) < 0 \quad [\because e^x > 0]$



$\therefore \; f'(x) < 0$ for $x \in (-3, 1)$

So, $f(x)$ is decreasing on $(-3, 1)$.

102. *(a)* The equation of curve is $y = \frac{2}{x^2}$

$$\therefore \quad \frac{dy}{dx} = \frac{-4}{x^3}$$

$$\therefore \text{ Slope of normal at } (x = 1) = \frac{-1}{\left.\frac{dy}{dx}\right|_{x=1}}$$

$$= \frac{-1}{\left(-\frac{4}{(1)^3}\right)} = \frac{1}{4}$$

Now, when $x = 1$, $y = \frac{2}{(1)^2} = 2$

$\therefore$ Equation of normal is given by

$$y - 2 = \frac{1}{4}(x - 1)$$

$\Rightarrow \quad 4y - 8 = x - 1$

$\Rightarrow \quad 4y - x - 7 = 0$

103. *(c)* We have,

$$f(x) = x^2 - x = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4}$$

Now, we know that

$$\left(x - \frac{1}{2}\right)^2 \geq 0 \Rightarrow \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} \geq -\frac{1}{4}$$

$$\Rightarrow \quad f(x) \geq -\frac{1}{4}$$

$\therefore$ Minimum value of $f(x)$ is $-\frac{1}{4}$

104. *(a)* Let, $I = \int 3x^2(x^3+1)^{10} dx$

Put $x^3 + 1 = t \Rightarrow 3x^2 dx = dt$

$$\therefore \quad I = \int t^{10} dt = \frac{t^{11}}{11} + C = \frac{(x^3+1)^{11}}{11} + C$$

105. *(e)* Let $I = \int \frac{2x + \sin 2x}{1 + \cos 2x} dx$

$$= \int \frac{2x + 2\sin x \cos x}{2\cos^2 x} dx$$

$$= \int (x \sec^2 x + \tan x)\, dx$$

$$= \int \underset{\text{I}}{x} \underset{\text{II}}{\sec^2 x}\, dx + \int \tan x\, dx$$

$$= x(\tan x) - \int (\tan x)\,.1\, dx + \int \tan x\, dx$$

$$= x \tan x + C$$

106. *(d)* Let $I = \int \frac{1}{x^2 - 25} dx = \int \frac{1}{x^2 - (5)^2} dx$

$$= \frac{1}{2 \times 5} \log \left| \frac{x-5}{x+5} \right| + C$$

$$\left[\because \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C\right]$$

$$= \frac{1}{10} \log \left| \frac{x-5}{x+5} \right| + C$$

107. *(a)* Let $I = \int \frac{1}{x(\log x)} dx$

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$\therefore \quad I = \int \frac{1}{t} dt = \log|t| + C = \log|\log x| + C$$

108. *(e)* Let $I = \int e^x \sec x(1 + \tan x) dx$

$$= \int e^x (\sec x + \sec x \tan x)\, dx$$

$$= \int \underset{\text{I}}{e^x} \underset{\text{II}}{\sec x}\, dx + \int e^x \sec x + \tan x\, dx$$

$$= \sec x(e^x) - \int e^x.(\sec x \tan x)\, dx + \int e^x \sec x \tan x\, dx$$

$$= e^x \sec x + C$$

109. *(d)* Let $I = \int \frac{1}{x + \sqrt{x}} dx = \int \frac{1}{\sqrt{x}(\sqrt{x}+1)} dx$

Put $\sqrt{x} + 1 = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\therefore \quad I = 2\int \frac{1}{t} dt = 2\log|t| + C = 2\log|\sqrt{x}+1| + C$$

110. *(a)* Let $I = \int \sec^2(5x-1)\, dx = \frac{1}{5}\tan(5x-1) + C$

$\left[\because \text{If } \int f(x)\,dx = F(x) + C, \text{ then } \int f(ax+b)\,dx = \frac{1}{a} F(ax+b) + C\right]$

111. *(b)* Let $I = \int_0^{\pi/2} \frac{1}{1 + \cot^4 x} dx$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx \quad \ldots \text{(i)}$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sin^4(\pi/2 - x)}{\sin^4(\pi/2 - x) + \cos^4(\pi/2 - x)} dx$$

$$\left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx \quad \ldots \text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} 1\, dx \Rightarrow 2I = [x]_0^{\pi/2}$$

$$\Rightarrow \quad 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

112. *(b)* Let $I = \int_{-10}^{10} (0.0002x^3 - 0.3x + 20)\, dx$

$$= \int_{-10}^{10} (0.0002x^3 - 0.3x)\, dx + 20\int_{-10}^{10} dx$$

$= I_1 + 20\,[x]_{-10}^{10}$, where

$$I_1 = \int_{-10}^{10} (0.0002x^3 - 0.3x)\, dx$$

$$= I_1 + 20\,[10 - (-10)] = I_1 + 400$$

Now, Let $f(x) = 0.0002x^3 - 0.3x$

$$\Rightarrow \quad f(-x) = 0.0002(-x)^3 - 0.3(-x)$$

$$= -0.0002x^3 + 0.3x$$

$$= -(0.0002x^3 - 0.3x) = -f(x)$$

$\therefore$ $f(x)$ is a odd function

$$\therefore \quad I_1 = \int_{-10}^{10} (0.0002x^3 - 0.3x)\, dx = 0$$

$$\left[\because \int_{-a}^{a} f(x)\, dx = 0, \text{ when } f(x) \text{ is odd}\right]$$

$$\therefore \quad I = I_1 + 400 = 400$$

113. *(b)* We have,

$$x = 3\cos\theta,\ y = 2\sin\theta$$

$$\therefore \quad \frac{x}{3} = \cos\theta \text{ and } \frac{y}{2} = \sin\theta$$

On squaring and then adding, we get

$$\frac{x^2}{9} + \frac{y^2}{4} = \cos^2\theta + \sin^2\theta$$

$$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{4} = 1$$

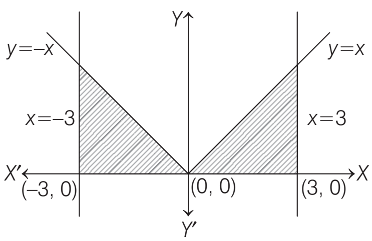
Which is equation of ellipse.

$\therefore$ Area enclosed $= \pi(3)(2)$ sq units.

$= 6\pi$ sq units.

[Area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab sq unit]

114. *(d)*



Required area $= 2\int_0^3 y\,dx = 2\int_0^3 x\,dx$

$$= 2\left[\frac{x^2}{2}\right]_0^3 = 9 \text{ sq unit}$$

115. *(b)* Let $I = \int_e^{e^2} \frac{1}{x}\,dx$

$$= [\log|x|]_e^{e^2} = \log e^2 - \log e$$

$$= 2\log e - \log e = 2 - 1 = 1$$

116. *(d)* Let $I = \int_{-3}^{3} |x+2|\,dx$

$$= \int_{-3}^{-2} |x+2|\,dx + = \int_{-2}^{3} |x+2|\,dx$$

$$= -\int_{-3}^{-2} (x+2)\,dx + \int_{-2}^{3} (x+2)\,dx$$

$$\left[\because |x| = \begin{cases} -x, & x < 0 \\ x, & x \ge 0 \end{cases}\right]$$

$$= -\left[\frac{x^2}{2} + 2x\right]_{-3}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{3}$$

$$= -\left[\left(\frac{4}{2} - 4\right) - \left(\frac{9}{2} - 6\right)\right] + \left[\left(\frac{9}{2} + 6\right) - \left(\frac{4}{2} - 4\right)\right]$$

$$= -\left[-2 + \frac{3}{2}\right] + \left[\frac{21}{2} + 2\right] = \frac{1}{2} + \frac{21}{2} + 2 = 13$$

117. *(a)* Given, differential equation

$$\frac{d^2y}{dx^2} + \sqrt{x^2 + \left(\frac{dy}{dx}\right)^{3/2}} = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\sqrt{x^2 + \left(\frac{dy}{dx}\right)^{3/2}}$$

$$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right)^2 = x^2 + \left(\frac{dy}{dx}\right)^{3/2}$$

$$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right) - x^2 = \left(\frac{dy}{dx}\right)^{3/2}$$

$$\Rightarrow \quad \left(\left(\frac{d^2y}{dx^2}\right)^2 - x^2\right)^2 = \left(\frac{dy}{dx}\right)^3$$

$$\Rightarrow \quad \left(\frac{d^2y}{dx^2}\right)^4 - 2x^2\left(\frac{d^2y}{dx^2}\right)^2 + x^4 = \left(\frac{dy}{dx}\right)^3$$

$\therefore$ Order = 2 and degree = 4

118. *(e)* We have, $\quad xy' + y = x^2$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{1}{x}y = x$$

This is linear differential equation

$$\therefore \quad \text{IF} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

$\therefore$ Solution of differential equation is given by

$$y.x = \int x.x\,dx + C$$

$$xy = \frac{x^3}{3} + C \quad \Rightarrow \quad y = \frac{x^2}{3} + \frac{C}{x}$$

119. *(c)* We have,

$$3xy' - y = 1 + \log x$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{1}{3x}y = \frac{1 + \log x}{3x}$$

$$\therefore \text{IF} = e^{\int -\frac{1}{3x}dx} = e^{-\frac{1}{3}\log x} = e^{\log(x)^{-1/3}} = (x)^{-1/3}$$

120. *(a)* We have,

$$y = \frac{A}{x} + B \quad \Rightarrow \quad xy = A + Bx$$

On differentiating both the sides with respect to x, we get

$$\Rightarrow \quad x\frac{dy}{dx} + y = 0 + B$$

On differentiating again, we get

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = 0$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$$